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Contributions to the Journal

The ACI JOURNAL technical pages are a medium for reporting what is of current interest in the concrete field; in research, design, construction, maintenance, or manufacturing. Members and readers are urged to "have their say" as a means of sharing knowledge and experience; time and changing interests demand that new names and new ideas take their places beside the old. Contributions describing new materials and techniques or new applications of familiar practices are welcomed by the Institute.

To guide the flow of material the Board of Direction has assigned to the Technical Activities Committee the selection of papers, committee reports, discussions, and other contributions for JOURNAL publication. To assist in their technical appraisal a corps of experts in various segments of the field volunteer their services. From the technical standpoint, acceptable contributions present original material for improvement of design, manufacturing, or construction; confirm, revise, or upset established ideas or practices; or review, digest, or arrange accumulated experience for more ready use.

General considerations by TAC are whether the contribution will be of interest and service, will fit into a well-balanced publications schedule, and can be published at reasonable expense to the Institute.

To expedite appraisal and publication, if accepted, contributors are requested to furnish glossy prints or inked tracings of illustrations accompanying manuscripts submitted for consideration. Further details of form and physical arrangement of contributions begin on p. 14, 1955 *ACI Directory*. Separate prints of the "publications policy" are available free upon request to the Secretary.

Manuscripts of papers, discussions, and reports
should be sent in triplicate to:

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AMERICAN CONCRETE INSTITUTE
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In ACI Journal next month

"Guide for Painting Concrete," to be reported by ACI COMMITTEE 616 in the March JOURNAL, describes types of paints, other than portland cement paint, commonly used on concrete. Procedures for preparing surface, selecting and applying paint, and repainting are reviewed. A list of federal specifications for paints suitable for concrete is included. Dampproofing and waterproofing are briefly discussed.

JOLDEAN MORROW and I. M. VIEST are co-authors of "Shear Strength of Reinforced Concrete Frame Members Without Web Reinforcement." Experimental and analytical studies reported by the pair indicate how the strength of a reinforced concrete frame member without web reinforcement is affected by shear through formation of diagonal tension cracks.

Development of the jet aircraft engine is changing not only the future of aviation, but also affecting many tangential fields, including the concrete industry, according to HERMAN G. PROTZE, author of "Structural Refractory Concrete." He considers theoretical and practical problems in the construction of jet engine test cell exhaust structures, presenting methods for installation of durable structural-refractory lightweight aluminous cement concretes.

The so-called floating slab foundation, used extensively for house construction, is applied to the foundation of a typical one-story wood-frame school building in "Design of Floating Slab Foundation" by NORMAN B. GREEN. He develops a method for stress analysis and design of a floating or monolithic type of slab foundation that supports edge loads. Cost savings are cited.

"Responsibilities of an Inspector," prepared by GRANT BLOODGOOD and LEWIS M. TUTHILL as a lecture for Bureau of Reclamation inspectors attending concrete and earth control courses, is to be published as part of the work of Committee 611. The committee recognizes that the inspector's effectiveness depends upon serious purpose and support of management behind him.

WALTER E. RILEY demonstrates the applicability of the moment distribution method when combined with principles of superposition to the practical analysis of continuous symmetrical arches on flexible piers in his paper "Analysis of Continuous Arches on Flexible Piers."

Title No. 53-39

Proposed

Recommended Practice for Design of Concrete Pavements*

Reported by ACI Committee 325

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SYNOPSIS

Recommendations are presented for the design of rigid concrete pavements and bases based on practice proved successful in the United States. It offers comprehensive directions to design rigid airport and highway pavements or bases for conditions of climate, traffic, available construction materials and equipment, and construction methods of the United States. It includes recommendations for soil foundations, selection of slab dimensions, joints, and details, for reinforced or nonreinforced pavement. Recommendations for design of cement-stabilized bases, continuously reinforced pavement, prestressed pavement, and rigid resurfacings are not included since their use has not yet developed a generally applicable practice. Design practices of all principal state and federal agencies concerned with paving have been reviewed and related within the limits recommended.

CHAPTER 1—INTRODUCTION

100—General notation

A_s = area of steel required per ft width of pavement or joints, sq in.
E_c = modulus of elasticity of concrete, psi
E_s = modulus of elasticity of steel, psi
F = coefficient of friction between slab and subgrade
f_b = allowable concrete bearing stress, psi
f_b' = ultimate bearing stress of concrete, psi
f_c' = ultimate compressive strength of concrete, psi
f_s = allowable tensile stress in steel, psi
f_t' = ultimate tensile strength of concrete at time of cracking, psi

*Title No. 35-39 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 8, Feb. 1957, *Proceedings* V. 53. Separate prints are available at 75 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than June 1, 1957. Address P. O. Box 4754, Redford Station, Detroit 19, Mich.

This report was submitted to letter ballot of the committee which consists of 17 members; 13 members returned their ballots of whom 11 have voted affirmatively and 2 negatively. It is released by the Standards Committee for publication and discussion.

MR = modulus of rupture, psi
f_t = allowable tensile stress in concrete, psi
FS = factor of safety
G = modulus of support of the concrete mass, lb per cu in.
h = pavement thickness, in.
I = moment of inertia, in.^{4}
k = modulus of foundation support to slab, lb per cu in.
L = length of slab between joints, ft
L' = distance between the joint in question and the nearest free joint or edge, ft
p = subgrade support reaction under the slab, psi
P = wheel load, lb
a = radius of a circular area equal to that of the actual area of contact of a single tire with pavement, in. For dual tires, a is the radius of a circle of area equal to the sum of the areas of contact of the two tires plus the area between them, assuming that the contact area of each is circular.
l = radius of relative stiffness, in.
u = allowable bond stress between concrete and steel, psi
w = weight of the pavement slab, psf
μ = Poisson's ratio (for concrete assume $\mu = 0.15$)
Z = slab deflection, positive downward, in.
a_1 = $a\sqrt{2}$

101—General scope

This report presents recommended practice for the structural design of concrete pavements for highways and airports. It includes recommendations for subgrades and subbases in relation to their effect on the structural design of the pavement. Recommendations are presented for the design of slab dimensions, for the design of joints and joint details, and for the design of steel reinforcement. The report is intended for use in conjunction with "Proposed Specifications for Concrete Pavements and Concrete Bases" (ACI 617).* It covers both nonreinforced and reinforced pavements but offers no recommendations for continuously reinforced pavements nor for concrete resurfacings.

102—General definitions

Concrete—A mixture of cement, fine aggregate, coarse aggregate, and water, which may include an approved admixture.

Concrete pavement—A rigid slab or series of slabs of portland cement type concrete, cast on a native soil subgrade or on a prepared subbase, generally exposed to weather, and designed to support traffic loads.

Crack—A permanent fissure or open seam within a concrete pavement at which tensile stress in the concrete has exceeded the tensile strength of the concrete.

Deformed bar—A reinforcing bar conforming to "Specifications for Minimum Requirements for the Deformations of Deformed Steel Bars for Concrete Reinforcement" (ASTM A 305). Bars not conforming to these specifications should be classed as plain bars.

Joint—The constructed junction of adjacent slabs interrupting the continuity of the pavement.

Reinforcement—Steel embedded in a concrete pavement to insure intimate contact of adjacent slab faces at cracks.

Slab—For design purposes, a monolithic portion of concrete pavement,

*To be published in a forthcoming ACI JOURNAL.—EDITOR.

bounded by joints or edges, within which continuity of tensile stress in the concrete is possible.

Slab length—Slab length in an uncracked pavement is the distance between adjacent transverse joints of any kind.

Subbase—Subbase, when used under a rigid pavement, refers to a relatively thin layer of selected or specified materials of a prescribed thickness placed between the subgrade and rigid pavement for such purposes as improvement of drainage, reduction in frost damage, prevention of pavement pumping, or moisture control of high volume change subgrade soils.*

Welded wire fabric—A two-way reinforcement system, fabricated from cold-drawn steel wire, having parallel longitudinal wires welded at regular intervals to parallel transverse wires and conforming to "Specifications for Welded Steel Wire Fabric for Concrete Reinforcement" (ASTM A 185 and AASHO M 55).

103—Reference specifications cited in this recommendation

(a) The standard of the American Concrete Institute listed below (with serial designation including the year of latest revision) is declared to be a part of this recommendation, the same as if fully set forth elsewhere herein: "Proposed Specifications for Concrete Pavements and Concrete Bases" (ACI 617).

(b) Further references which discuss the purposes and intentions of this recommendation and which set forth research data and theoretical considerations and derivations justifying this recommendation are the following separately published reports of the four subcommittees of Committee 325:

1. Subcommittee I, ACI Committee 325, "Considerations for Construction of Subgrades and Subbases for Rigid Pavement," ACI JOURNAL, Aug. 1956, *Proc.* V. 53, pp. 145-155.

2. Subcommittee II, ACI Committee 325, "Considerations in the Selection of Slab Dimensions," ACI JOURNAL, Nov. 1956, *Proc.* V. 53, pp. 433-454.

3. Subcommittee III, ACI Committee 325, "Structural Design Considerations for Pavement Joints," ACI JOURNAL, July 1956, *Proc.* V. 53, pp. 1-28.

4. Subcommittee IV, ACI Committee 325, "Design Considerations for Concrete Pavement Reinforcement for Crack Control," ACI JOURNAL, Oct. 1956, *Proc.* V. 53, pp. 337-362.

104—Materials and tests

All materials to be used in constructing the subgrade, subbase, and the concrete pavement should conform to the requirements of "Proposed Specifications for Concrete Pavements and Concrete Bases" (ACI 617).

CHAPTER 2—CONCRETE PROPERTIES AND ALLOWABLE STRESSES

201—Concrete properties

Concrete properties as specified in "Proposed Specifications for Concrete Pavements and Concrete Bases" (ACI 617) should be the minimum required.

*This definition of subbase, for purposes of pavement design, includes both subbases and soil-aggregate base courses as defined for specification purposes in "Proposed Specifications for Concrete Pavements and Concrete Bases" (ACI 617).

202—Allowable unit stresses in concrete

(a) The unit stresses in pounds per square inch in concrete to be used in design of concrete pavements should not exceed the following values:

$f_t = 0.50$ MR

$u = 0.10 f_c'$, but not more than 350 psi for deformed bars or welded wire fabric

$u = 0.045 f_c'$, but not more than 160 psi for plain bars

(b) The unit bearing stresses in pounds per square inch in concrete under dowels to be used in design of concrete should not exceed the values in Table 202(b).

TABLE 202(b)—ALLOWABLE UNIT BEARING STRESSES IN CONCRETE UNDER DOWELS

Dowel diameter, in.	f_b, psi
3/4	3200
7/8	3100
1	3000
1 1/8	2900
1 1/4	2800
1 3/8	2650
1 1/2	2500
2	2000

203—Allowable unit stresses in steel

The unit stresses in pounds per square inch in steel for use in concrete pavements should not exceed the following values:

Type and grade of steel	Minimum yield point, psi	f_s, psi
Billet and axle steel, structural grade	33,000	22,000
Billet and axle steel, intermediate grade	40,000	27,000
Rail steel or billet and axle steel, hard grade	50,000	33,000
Cold drawn wire	56,000	37,000

CHAPTER 3—SUBGRADES AND SUBBASES

301—Definitions

Drainage—Drainage is the control of water accumulations on or in subgrades or subbases as necessary to insure satisfactory performance of the pavement.

Filter course—A subbase graded so that underlying fine-grained subgrade soils are prevented from penetrating any appreciable distance upward into this layer thus preventing detrimental subgrade pumping.

Frost action—Frost action in soil is a phenomenon involving freezing, thawing, and movement of moisture in soil, and consequent changes in its physical properties.

Frost heave—Frost heave is the surficial expression of the volume changes which take place when (1) soil water is frozen and (2) ice lenses form within the zones of freezing.

Frost-susceptible soil—Material possessing sufficient permeability and a relatively high degree of capillarity such as silts and varved clays.

Pumping—Pumping in a concrete pavement is a process in which the subgrade soil and water form a suspension which is forced to the surface at the edges, cracks, and joints of the pavement under traffic loads.

Shrinkage and swelling—These phenomena represent one portion of the general category of volume change in soils and are caused by varying soil moisture contents.

302—Pavement pumping

Pumping occurs at or near joints and cracks and along the edges of concrete pavement, and may be recognized by soil stains on the pavement and soil-water slurries at pavement edges. Three factors are essential to pumping: (1) free water must be present beneath the slab; (2) axle loadings must be heavy and frequent; and (3) the subgrade soil must be capable of going into suspension. If any one of the foregoing factors is absent, pumping will not occur. Breaking of pavements will result from continued pumping.

The use of granular materials as relatively thin subbases, when designed as a filter course, the use of selected soils and granular materials as thicker subbases, or selected soils and granular materials stabilized with a cementing agent are effective methods of preventing pumping on new construction. Pumping may also be minimized through the use of adequately designed load-transfer devices which reduce pavement deflection and prevent faulting, and through the use of adequate reinforcement.

303—Subbases

(a) *Subbase functions*

1. The subbase for a rigid pavement is a compacted layer of planned thickness that is placed immediately under the pavement and on the subgrade to serve one or all of the following functions:

a. To provide uniform, stable, and permanent support for the pavement.
b. To prevent or reduce the damaging effects of frost action.
c. To improve drainage and, when properly drained, minimize the accumulation of water under the pavement.
d. To prevent pumping action at joints, cracks, and edges of pavement.
e. To reduce volume changes in the subgrade and to minimize the detrimental effects of such volume changes on the pavement.

2. Thicknesses greater than 6 in. are used to replace frost-susceptible subgrade soils, minimize the effect of volume change, and to provide increased subbase-pavement strength, in addition to preventing pumping.

(b) *Minimum load-supporting value*—Where economical, the finished subbase should have a load-supporting value such that when it is loaded in increments with a 30-in. diameter steel plate, it will develop a modulus of foundation reaction, k, of not less than 200 lb per cu in. The subgrade modulus, k, should be evaluated by loading tests made directly on the finished subbase and/or trial sections of the subbase constructed in both cut and fill areas and at the transition between cut and fill areas.

(c) *Requirements to prevent pumping*—Where the subgrade soil is capable of going into suspension, filter course subbases 3 to 6 in. thick will prevent pumping.

304—Effect of subbase and subgrade on pavement thickness design

The thickness of pavement required for given traffic loads and other conditions is not affected significantly by the presence or absence of a granular subbase layer of the usual thicknesses, but strong stabilized subbases may have an effect. The supporting power of the subgrade or combined subgrade and subbase does affect the thickness of pavement required, particularly where heavy wheel or axle loads are involved.

(a) *Corps of Engineers method*

1. Under normal conditions, or where freezing is moderate, or where the water table is deep, a 4 in. minimum thickness of subbase is used.

2. When frost action is anticipated, modifications in the design for normal conditions are made. With an F-4 subgrade soil, the worst from the standpoint of frost susceptibility, a total thickness of subbase and pavement equal to the depth of frost penetration is used. Subgrade soils of less severe frost susceptibility are associated with a design modification that allows for a reduction in strength due to the frost action. A reduction in the modulus of subgrade reaction is made based upon appropriate charts. A thickness of subbase is used equal to the thickness of the slab designed assuming the reduced k value.

(b) *Navy method*—The thickness of subbase to be used is dependent upon the results of loading tests made on the trial sections during the evaluation of the subgrade.

(c) *Portland Cement Association method*—A subbase is added to the pavement when nonuniform soil conditions exist or when detrimental volume changes or pumping are anticipated. The thickness of subbase is arbitrary and is recommended as one-half to one-third of the depth of frost penetration, 4 to 12 in. in cases of high volume change soils, and 4 to 6 in. in the case of anticipated pumping action.

(d) *Civil Aeronautics Administration method*—A constant subgrade modulus is assumed as a result of the provision of variable subbase thicknesses which depend on wheel loading, subgrade soil, drainage conditions, and frost penetration.

CHAPTER 4—SLAB DIMENSIONS

400—Notations

y = longitudinal direction of pavement
x = horizontal direction normal to y
$t(y)$ = corresponding wheel load stresses
$t(x)$ = corresponding wheel load stresses
a_e = semi-major axis of tire contact area, in.
b_e = semi-minor axis of tire contact area, in.
B,R,C,S = space and area coefficients

401—Slab lengths

(a) *Nonreinforced concrete pavements*

1. The spacing of contraction cracks in nonreinforced concrete pavements

without transverse joints other than construction joints placed at the end of each day of work may be predicted by the formula:

$$L = \frac{2 f_t'^*}{F} \text{ in ft} \quad \ldots\ldots (1)$$

2. Warping stresses may be assumed constant for 6-in. slabs longer than 18 ft and for 9-in. slabs longer than 24 ft. For longer slabs, maximum warping stresses in 9-in. slabs may be taken as 50 percent greater than those in 6-in. slabs. For shorter slabs, the warping stress is greater in 6-in. slabs. Increases in k values result in increased warping stresses in short slabs. See Fig. 401(a)1 and 401(a)2 for design data.†

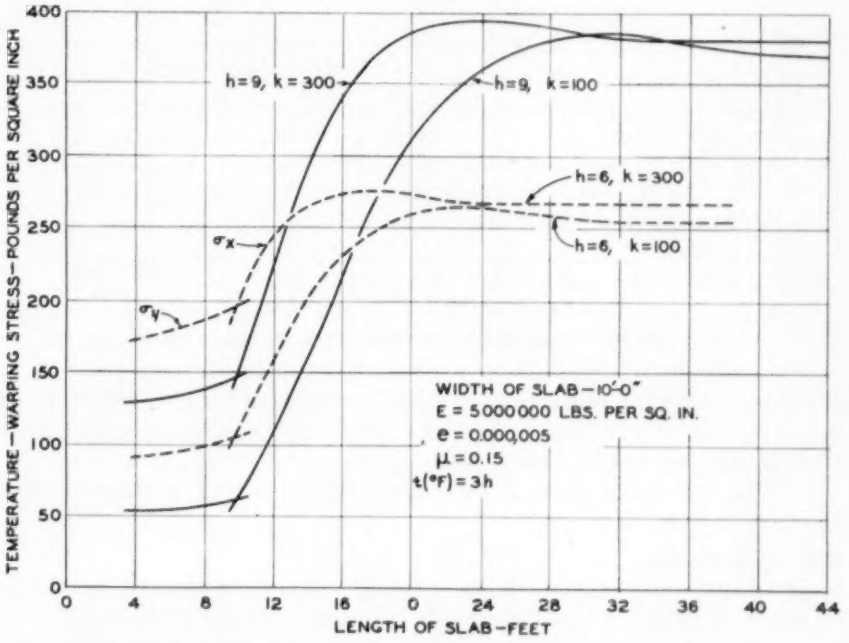


Fig. 401(a)1—Temperature-warping stresses—interior of slab

3. For nonreinforced concrete pavements, under average conditions a transverse joint spacing of 20 ft is recommended to prevent intermediate cracking. For unusual conditions this spacing may be varied to 15 or 25 ft.

Where load transfer at joints is to depend wholly or partially upon aggregate interlock, expansion joints intermediate between structures except for special conditions should be eliminated.

(b) *Reinforced concrete pavements*

1. Since effective crack control can be maintained by the addition of

*Unless the curing conditions prevent early shrinkage, f_t' will be very low, often based on an age less than 24 hr.
†Warping stresses have never been measured directly. Theoretical predictions have neglected and strain measurements have included the effect of creep which reduces these stresses.

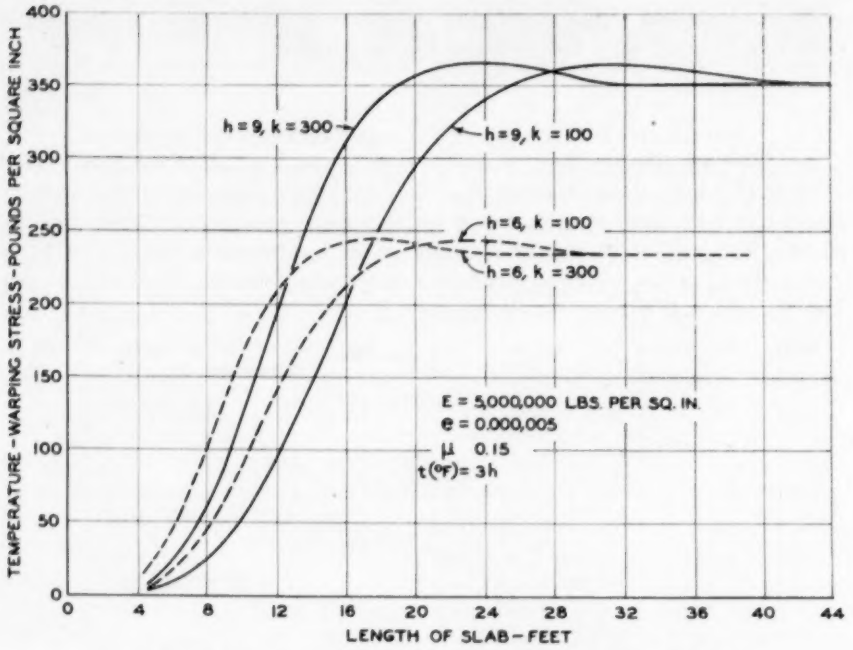


Fig. 401(a)2—Temperature-warping stresses—edge of slab

distributed steel, the economical length of a reinforced concrete slab depends upon: (a) cost of distributed steel; (b) initial cost of joints, including load-transfer devices; and (c) cost of joint maintenance.

2. The commonly used joint spacing for reinforced pavement varies between 40 and 100 ft.

402—Slab widths

(a) Traffic requirements make a 12-ft minimum lane-width necessary for maximum capacity of mixed (passenger and commercial) traffic.

(b) For highway pavement, to prevent irregular longitudinal cracking, a maximum spacing of 12 ft between longitudinal joints is recommended.

(c) For airport paving, to prevent irregular longitudinal cracking, longitudinal construction joints should be spaced at not more than 25 ft. For pavements 10 in. or less in thickness, an intermediate weakened plane joint of the dummy type should be required.

403—Slab thickness

(a) *Basic concepts for design*

1. Warping stresses due to differential temperatures between the top and bottom of the slab are daily occurrences. Their magnitude is high and they, combined with traffic stresses, can and do produce transverse cracking in the pavement.

2. Wheel loads also produce high bending stress, especially along the edges of the slab and near the corners, but these are of lesser amount in the center of the slab. Excessive bending can produce accentuated pumping.

3. Load transfer across joints or cracks helps to relieve joint, edge, and corner stresses.

4. Distributed steel may not prevent cracking, but it does hold the slab together at the cracks so that faulting of the pavement at these cracks is avoided. When cracks occur, the high warping stress which formerly existed in the vicinity of the crack becomes negligible.

5. Subgrade friction in slabs longer than suitable for unreinforced pavements should be resisted by longitudinal steel, which should be sufficiently strong to prevent its failure due to excessive tension.

6. Concrete suffers fatigue under repeated heavy loading and the importance of this fact must at least be considered in any theory of pavement design. However, if the maximum stress is no greater than half the modulus of rupture, fatigue will not be a factor in design. It is recommended that the pavement thickness be so designed that under the assumed maximum wheel load the stress will be restricted to half the modulus of rupture.

7. Concrete usually fails by tension produced either directly or through bending.

8. When designing nonreinforced slabs, warping stresses as well as traffic stresses must be considered, and the joints should be spaced so as to prevent intermediate cracks from forming due to these combined stresses.

9. When designing reinforced concrete pavement, warping stresses need not be considered in addition to load stresses.

(b) *Design theory*

1. The accepted theory (developed by Westergaard) involves one important assumption that is expressed in the simple formula:

$$p = k\,Z \qquad (2)$$

Obviously, Eq. (2) is not applicable to nonuniform or spot support of the slab. The theory does not pertain to the case of unsupported corners and edges of the slab. The modulus of foundation support k is assumed to be constant over the entire paved area and during the life of the pavement.

2. For finding the modulus of foundation support, k, it is considered that a satisfactory procedure is by the use of loading tests with a 30-in. diameter steel plate. This test should be made, whenever possible, on trial sections of variable thicknesses of compacted subbase or of compacted subgrade if the subgrade material is suitable for a subbase. The control of moisture and density of the subbase is of paramount importance in this connection.

3. Wheel loads produce flexural stresses in a concrete pavement. The pavement fails under combined loading and warping conditions which cause a flexural stress equal to or greater than its flexural strength or modulus of rupture. The factor of safety, FS, against overloading of the slab is:

$$FS = \frac{MR}{f_t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

In this expression, MR is the modulus of rupture (psi) and f_t is the maximum principal bending stress (psi). A minimum factor of safety of 2 should be provided, except for certain airport pavements where the safety factor may be less, depending on the design wheel load.

4. If the pavement is designed so that the maximum stress is no greater than half the modulus of rupture, fatigue will not enter into the design problem as an important factor. The pavement thickness should be so designed that under the assumed maximum wheel load the stress will be restricted to roughly half the modulus of rupture. The assumed wheel load should be greater than the legal load limit and it is well to assume an additional 10 percent for impact.

5. Separate design criteria are applicable for concrete airport and highway pavements. For airport pavements, computations of wheel load stresses for any given load and for different thicknesses of concrete have reference to loading at an interior point of the slab. In the case of concrete highway pavements the computed stresses have reference to corner slab loading.

(c) *Design application for airport pavement**

1. Single tire loading.

The choice of coordinate axes is arbitrary. Let y be in the longitudinal direction of the concrete highway or runway pavement and let the x direction be the normal horizontal to that of the y direction. Let the corresponding wheel load stresses be designated as $t(y)$ and $t(x)$. The tire contact area tends to be of elliptical shape, the major axis having a length of from 1.5 to 2.0 times the length of the minor axis. Let a_e and b_e denote the semi-major and semi-minor axes, respectively. Then the Westergaard expression for $t(x)$ and $t(y)$ [Poisson's ratio assumed to be 0.15] when reduced to simplest form is the following:

$$\left.\begin{matrix} t(y) \\ t(x) \end{matrix}\right] = \frac{P}{h^2}\left[0.316 \log_{10} \frac{\dfrac{E_c h^3}{k (a_e + b_e)^4}}{2} \mp (0.203) \frac{(a_e - b_e)}{(a_e + b_e)}\right] \quad \ldots\ldots\ldots (4)\dagger$$

In these expressions, the minus sign on the right corresponds to $t(x)$ and the plus sign to $t(y)$.

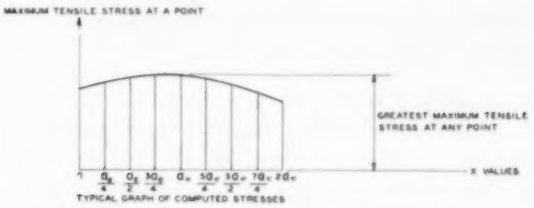
2. Dual tire loading

For dual tire loading, the following expressions are used, assuming that Poisson's ratio for concrete is 0.15:

$$\left.\begin{matrix} t(x) \\ t(y) \end{matrix}\right] = \frac{3P}{2\pi h^2}\left[(1.15)(B + R) \mp (0.425)(C + S)\right] \quad \ldots\ldots\ldots (5)*$$

*See Appendix A402.

†Westergaard, H. M., "New Formulas for Stresses in Concrete Pavements for Airfields," *Transactions*, ASCE, V. 113, 1948, pp. 425-444, and "Stress Concentrations in Plates Loaded over Small Areas," *Proceedings*, ASCE, V. 68, Apr. 1942, pp. 509-534.


MAXIMUM TENSILE STRESS AT A POINT
GREATEST MAXIMUM TENSILE STRESS AT ANY POINT
X VALUES
TYPICAL GRAPH OF COMPUTED STRESSES

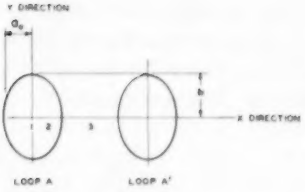


Fig. 403(c)—Dual tire loading on concrete pavement

The coefficients B, R, C, and S are "space and area coefficients" and are determined numerically by the spacing of loads and the shapes and sizes of the loaded areas. For elliptical tire contact areas, $C = 0$ and $B = 0.1159$. With these values the foregoing expressions may be written in the following form:

$$\left.\begin{matrix} t(x) \\ t(y) \end{matrix}\right] = \frac{3P}{2\pi h^2}\left[(1.15)(0.1159 + R) \mp (0.425S)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)^*$$

With reference to Fig. 403(c), the stresses at points (such as *1* and *2*) in the concrete below load A are *within* ellipse A and *outside* ellipse A'. The point *3* is outside both ellipses. The expressions for the coefficients, R and S, are different for points inside and outside the ellipse. For example, the stress at point *2*, under load A, is caused by the superposition of loads A and A' at this point. For the stress at points such as *1* and *2* due to load A alone, the following expressions are used for computing the numerical values of R and S:

$$\left.\begin{aligned} R &= 0.5000 + \log_e\left[\frac{2L}{a_e + b_e}\right] - \frac{x^2}{a_e^2 + a_e b_e} - \frac{y^2}{a_e b_e + b_e^2} \\ S &= \frac{b_e - a_e}{a_e + b_e} - \frac{2a_e b_e}{(a_e + b_e)^2}\left[\frac{x^2}{a_e^2} - \frac{y^2}{b_e^2}\right] \end{aligned}\right\} \quad \ldots\ldots\ldots (7)$$

For point *1*, x and y are both zero since this point is the origin. For point *2*, x has a finite value but y is zero since the point is on the x axis.

The stresses in concrete at interior points of ellipse A due to the load A', the stresses at points such as *3* which is outside both ellipses, and the stresses

*Westergaard, H. M., *op. cit.*

within ellipse A' due to load A require for their computation the use of the following expressions for R and S:

$$\left.\begin{aligned} R &= 0.5000 + \log_e \frac{2l}{x + \sqrt{x^2 - a_e^2 + b_e^2}} - \frac{x}{x + \sqrt{x^2 - a_e^2 + b_e^2}} \\ S &= \frac{b_e^2 + a_e^2}{b_e^2 - a_e^2} - \frac{4b_e^2\, x}{\left[x + \sqrt{x^2 - a_e^2 + b_e^2}\right](b_e^2 - a_e^2)} \end{aligned}\right\} \quad \ldots\ldots (8)$$

In the foregoing expressions for R and S, b_e and a_e are the semi-major and semi-minor axes of the ellipses, respectively; the two elliptic tire prints being identical with respect to size and shape; l is the radius of relative stiffness of the slab [see Table 403(d)].

$$l = \sqrt[4]{\frac{E_c h^3}{12\,(1 - \mu^2)\, k}} \quad \ldots\ldots (9)$$

For concrete pavement, Poisson's ratio, μ, is usually taken as 0.15.

TABLE 403(d)—VALUES FOR *l* CORRESPONDING TO DIFFERENT VALUES OF *k* AND *h

k, lb per cu in.	50	100	150	200	250	300	350	400	500
h, in.	Radius of relative stiffness, *l*								
6	34.84	29.30	26.47	24.63	23.30	22.26	21.42	20.72	19.59
6.5	36.99	31.11	28.11	26.16	24.74	23.64	22.74	22.00	20.80
7	39.11	32.89	29.72	27.65	26.15	24.99	24.04	23.25	21.99
7.5	41.19	34.63	31.29	29.12	27.54	26.32	25.32	24.49	23.16
8	43.23	36.35	32.85	30.57	28.91	27.62	26.58	25.70	24.31
8.5	45.24	38.04	37.37	31.99	30.25	28.91	27.81	26.90	25.44
9	47.22	39.71	35.88	33.39	31.58	30.17	29.03	28.08	26.55
9.5	49.17	41.35	37.36	34.77	32.89	31.42	30.23	29.24	27.65
10	51.10	42.97	38.83	36.14	34.17	32.65	31.42	30.39	28.74
10.5	53.01	44.57	40.28	37.48	35.45	33.87	32.59	31.52	29.81
11	54.89	46.16	41.71	38.81	36.71	35.07	33.75	32.64	30.87
11.5	56.75	47.72	43.12	40.13	37.95	36.26	34.89	33.74	31.91
12	58.59	49.27	44.52	41.43	39.18	37.44	36.02	34.84	32.95
12.5	60.41	50.80	45.90	42.72	40.40	38.60	37.14	35.92	33.97
13	62.22	52.32	47.27	43.99	41.61	39.75	38.25	36.99	34.99
13.5	64.00	53.82	48.63	45.26	42.80	40.89	39.35	38.06	35.99
14	65.77	55.31	49.98	46.51	43.98	42.02	40.44	39.11	36.99
14.5	67.53	56.78	51.31	47.75	45.16	43.15	41.51	40.15	37.97
15	69.27	58.25	52.63	48.98	46.32	44.26	42.58	41.19	38.95
15.5	70.99	59.70	53.94	50.20	47.47	45.36	43.64	42.41	39.92
16	72.70	61.13	55.24	51.41	48.62	46.45	44.70	43.23	40.88
16.5	74.40	62.56	56.53	52.61	49.75	47.54	45.74	44.24	41.84
17	76.08	63.98	57.81	53.80	50.88	48.61	46.77	45.24	42.78
17.5	77.75	65.38	59.08	54.98	52.00	49.68	47.80	46.23	43.72
18	79.41	66.78	60.35	56.16	53.11	50.74	48.82	47.22	44.66
19	82.70	69.54	62.84	58.48	55.31	52.84	50.84	49.17	46.51
20	85.95	72.27	65.30	60.77	57.47	54.92	52.84	51.10	48.33
21	89.15	74.97	67.74	63.04	59.62	56.96	54.81	53.01	50.13
22	92.31	77.63	70.14	65.28	61.73	58.98	56.75	54.89	51.91
23	95.44	80.26	72.52	67.49	63.83	60.98	58.68	56.75	53.67
24	98.54	82.86	74.87	69.68	65.90	62.96	60.58	58.59	55.41

*E_c = 4,000,000 psi and μ = 0.15

Radius of relative stiffness = $l = \sqrt[4]{\frac{E_c(h^3)}{12(1 - \mu^2)\, k}} = 24.162 \sqrt[4]{\frac{h^3}{k}}$

The Westergaard expressions for place and area coefficients and for stresses in the slab utilize the first few terms of infinite series and are therefore applicable for only "points at or near the loaded area" (Westergaard). In the present case, dual tire loading, the expressions are applicable without appreciable error provided that the distance between the ellipses does not exceed the length of the major axis, which is $2b_e$ [Fig. 403(c)]. If the two tires are farther apart than this, it is much more accurate to use the influence chart and the graphical method of Pickett and Ray which is based on the exact and not the approximate procedure.

The computed stresses at points *1*, *2*, *3*, etc., of the lower diagram of Fig. 403(c) are plotted in the upper diagram of this figure against the stresses at these several points. Then a smooth curve, as shown in this figure, is drawn through these plotted points. It is preferable to plot the distances of the points *1*, *2*, *3*, etc., from the origin, in terms of a_e, the minor semi-axis of each tire footprint. The stress versus distance from the origin curve of the upper diagram has a maximum ordinate. In the diagram shown, this maximum is at a value of x that is slightly less than a_e. The position of the maximum stress necessarily changes with changes in the distance between the loaded elliptical areas, that is, the tire spacing.

(d) *Design application for highway pavement*

The formulas developed by Pickett are used and two conditions are considered:

1. *Protected corners*—Provision has been made to transfer at least 20 percent of the load from one slab corner to the other by some adequate mechanical means or by aggregate interlock. For this case

$$f_t = \frac{3.36P}{h^2}\left[1 - \frac{\sqrt{a/l}}{0.925 + 0.22\,a/l}\right] \qquad (10)$$

See Appendix A401 for design curves.

2. *Unprotected corners*—There is no adequate provision for load transfer and one corner must carry over 80 percent of the load.

$$f_t = \frac{4.2P}{h^2}\left[1 - \frac{\sqrt{a/l}}{0.925 + 0.22\,a/l}\right] \qquad (11)$$

See Appendix A401 for design curves.

In the two foregoing formulas

f_t = maximum tensile stress (psi) at top of slab and in the direction of the bisector of the corner angle

P = wheel load, lb (gross load plus impact). An impact allowance of 20 percent is recommended for pneumatic tires

h = thickness (in.) of a uniform-thickness concrete slab at a corner (or equivalent thickness of a thickened-edge slab)

CHAPTER 5—JOINTS, LOAD TRANSFER DEVICES, AND DETAILS

500—Notation

W = joint width opening, in.
d = diameter of dowel, in.
β = relative stiffness of the dowel and the concrete mass
L_2 = length of the primary bearing pressure wave on dowel, in.
L_T = length of tie-bar, in.
P_D = concentrated load acting downward on dowel at the center of a joint, lb
Q = perimeter of tie-bar, in.
S_b = bearing pressure on concrete under dowel at slab face, psi
A_s' = cross-section area of one tie-bar, sq in.
e = base of Napierian logarithms
x = distance along the dowel from slab face at the joint, in.
M_o = bending moment in dowel at slab face, in.-lb; $M_o = (P_D W)/2$
x_m = distance from slab face to point of maximum moment in dowel, in.

501—Definitions

(a) *Dowel*—A plain bar embedded in a concrete pavement to provide vertical shear resistance across a joint.

(b) *Tie bar*—A deformed bar embedded in a concrete pavement at a joint and designed to hold abutting slabs together.

(c) *Joints*

1. Expansion joints

As the term implies, the primary function of an expansion joint is to provide space for the expansion of the pavement and thereby prevent the development of compressive stresses of damaging magnitude. When installed between the pavement and a fixed structure, such as a bridge, an expansion joint serves also to prevent the pavement from exerting damaging pressures against the structure. Expansion joints also function as contraction joints.

2. Contraction joints

The purpose of a contraction joint is to limit tensile stresses to a safe value. Since contraction joints must be free to open, the continuity of the reinforcement, if reinforcement is present, is interrupted at the joints.

Basically, there are three types of contraction joints, namely, "groove joint," "plate joint," and "butt joint."

a. Groove joint—This joint is created by forming a groove in the upper portion of the pavement. Methods employed to form the groove consist of (1) temporary embedment of a suitable metal strip; (2) installation of a thin strip of premolded joint filler material to the required depth; and (3) sawing the pavement after the concrete has hardened.

In addition to the surface groove, a parting strip of wood, metal, or premolded material is sometimes provided at the bottom of the pavement, directly under the groove, to insure the formation of a vertical crack and to further encourage early cracking of the concrete at the joint.

b. Plate joint—This joint is created by erecting a separator or parting strip on the subgrade. The separator, which usually consists of a metal plate or a thin sheet of some other rigid, noncompressible material, serves to interrupt the continuity of the pavement. A groove is usually formed in the concrete immediately above the separator, to serve as a reservoir for sealing material.

c. Butt joint—This joint occurs where fresh concrete is placed against hardened concrete.

3. Hinge or warping joints

The term "hinge" or "warping joint" applies to any joint which permits hinge action, but no appreciable separation of the adjacent slabs. Hinge joints are intended primarily to relieve warping stresses. In contrast to contraction or expansion joints, appreciable changes in joint width are prevented by the installation of steel tie bars across the joint. In effect, a joint of this type acts simply as a hinge, that is, it merely permits the abutting slabs to undergo a certain amount of vertical angular displacement.

Butt joints become hinge joints when adjacent slabs are held together with tie bars.

4. Construction joints

The term applies to the joint necessitated by any prolonged stoppage of the concreting operations, such as the end of the day's work, equipment breakdown, or delay in delivery of materials.

5. Longitudinal joints

Because of the longitudinal cracking that has occurred extensively in highway pavements constructed with full-width slabs, it has become general practice to divide the pavement into lanes by longitudinal joints.

This joint may be an ordinary butt joint resulting from "lane-at-a-time" construction, or one created during full-width construction by embedding a metal or fiber plate in the concrete along the proposed line of the joint and so shaped as to create mechanical interlocking of the slab edges; or it may be created by weakening the pavement with a deeply formed groove, sawing, or with an embedded strip of premolded material. Adjacent lanes are usually kept from separating and faulting by properly spaced steel tie bars.

(d) *Mechanical load-transfer devices*

Some form of load transfer should be provided even though the edges and corners of the slabs are designed to carry the load without overstressing the concrete. Loads passing over a joint without load transfer may produce excessive deflections, resulting in permanent damage to the subgrade. This in turn may lead to excessive vertical displacement of slab ends and eventual faulting at joints, even though concrete stresses are not serious.

1. Mechanical load-transfer devices may be divided into two major classifications:

a. Those which develop resistance to shear, but little or no resistance to bending. Included are face-interlock types such as deformed plates and some proprietary makes of load-transfer devices.

b. Those which develop resistance to both shear and bending, of which the dowel bar is the most common. Most proprietary load-transfer devices employ this principle in their design.

2. Mechanical load-transfer devices should possess the following attributes:

a. They should be simple in design in order that they may be practical to install, and permit positive encasement by the concrete.

b. They should be capable of distributing the load stresses throughout the adjacent concrete in such a manner that these stresses will not exceed the allowable design value. In this respect, it is especially important that high localized stresses in the concrete at the joint face be prevented.

c. They should offer no material restraint to the opening of the joints at any time.

d. They should retain their mechanical stability under wheel-load weights and frequencies comparable to those for which the pavement itself has been designed.

e. They should be constructed in such a manner as to meet specified performance requirements relative to load-transfer capacity.

f. They should be resistant to corrosion.

3. The conventional round steel dowel is used to a greater extent than any other type of mechanical load-transfer device. For any given load and pavement condition, it is recommended that all other types of mechanical load-transfer devices be equivalent in load-transfer capacity to that of a round dowel of the dimensions determined in accordance with Sections 502(e) and 503(f).

(e) *Aggregate interlock*—The use of aggregate interlock to transfer loads across joints and to maintain mutual alignment of abutting slabs is generally associated with the construction of nonreinforced pavement and with the use of groove or sawed contraction joints. In order that aggregate interlock may be reasonably effective, the maximum opening of joints should not exceed 0.02 in. except in cases where the pavement is supported by an unusually strong foundation. But even with these precautions, experience indicates that aggregate interlock frequently is not satisfactory under a large volume of heavy truck traffic.

502—Highway pavements

(a) *Expansion joints*

The width of expansion space per joint and the total expansion space needed are governed by pavement design details, aggregates used, and local climatic conditions. For practical reasons, the maximum width is limited to 1 in. The usual width of opening is ¾ in. To provide more than 1 in. of expansion space at special locations, such as at bridges and railroad crossings, where it is desirable to limit compressive stresses to a safe value, it is recommended that two or three expansion joints of standard width be installed consecutively at intervals of approximately 20 ft. This need apply only in cases where expansion joints are omitted generally elsewhere in the pavement.

Mechanical load-transfer devices are necessary at expansion joints.

At expansion joints, the sliding ends of the dowels must be enclosed in a close-fitting metal cap or thimble to provide space for the movement of the dowel into the concrete during an expansion cycle. The space should be equivalent to the width of the expansion joint, plus ¼ in.

Recommended design details for expansion joints are shown in Fig. 502(a).

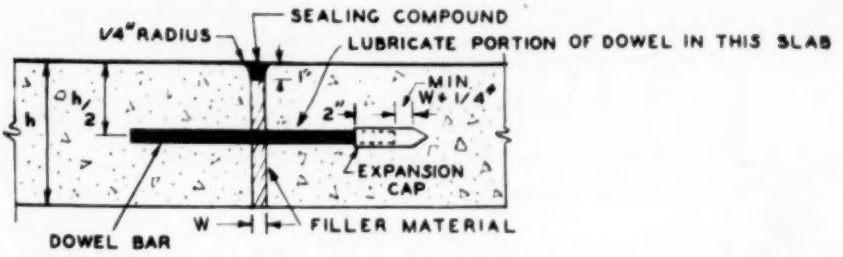


Fig. 502(a)—Recommended design details for highway transverse expansion joint

(b) *Contraction joints*

Recommended details for contraction joints are set forth in the following paragraphs and illustrated in Fig. 502(b).

1. Groove joint

This type of contraction joint is more widely used than any other. The depth of the surface groove should be not less than 1/6 nor more than ¼ the thickness of the pavement. The depth of the groove may be decreased to 1½ in. if a groove is also formed at the bottom of the pavement by installing a parting strip of suitable height on the subgrade, and in the same vertical plane as the surface groove. The depth of the groove plus the height of the parting strip should not exceed 1/3 the pavement thickness. The width of the groove should be at least twice the anticipated annual variation in joint

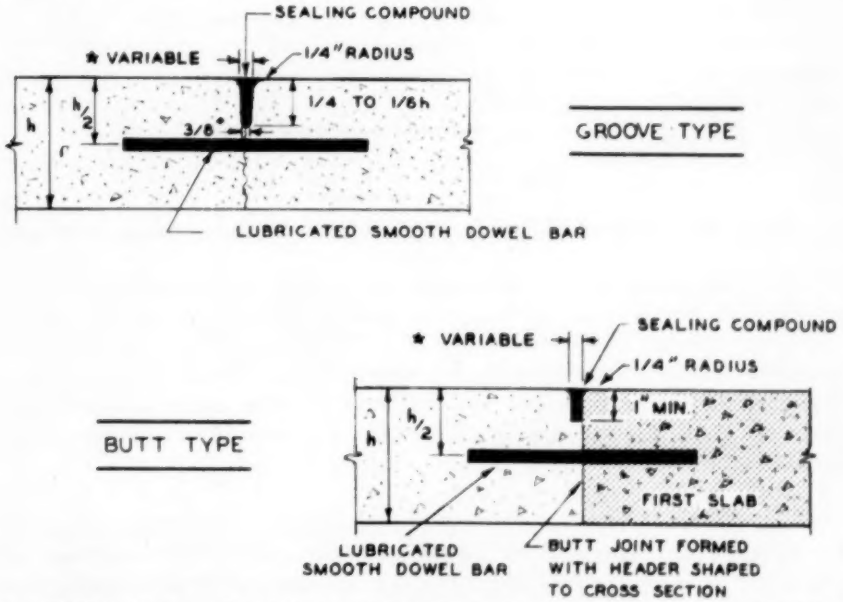


Fig. 502(b)—Recommended design details for highway transverse contraction joints

*Width at least twice annual variation in joint width but not less than ⅛ in.

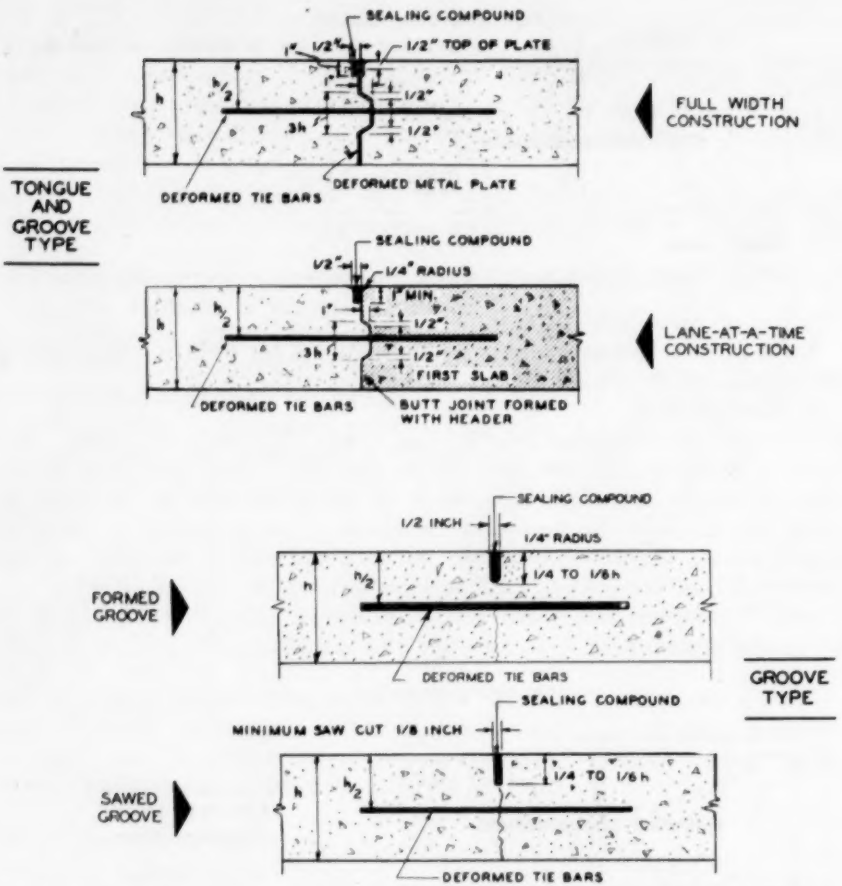


Fig. 502(c)—Recommended design details for highway longitudinal hinge or warping joints

NOTE: Crack in "formed groove" joint should extend above the tie bar.—EDITOR

width, but not less than 1/8 in. This is necessary to facilitate sealing of the joint and to insure that the joint will remain in a sealed condition.

Mechanical load-transfer devices are recommended under all conditions where the joint spacing exceeds 20 ft, and even at a lesser spacing where service conditions are severe.

2. Flat plate joint.

A thin transverse strip of noncompressible material is left in place to form this joint. The strip should be capped in such a manner that when the cap is removed it will form a groove adequate for retention of joint-sealing material. Load-transfer devices are recommended under all conditions of joint spacing.

3. Deformed plate joint

Recently, plates with horizontal deformations or corrugations have been used for the dual purpose of forming the joint and providing mechanical interlock. In the absence of conclusive performance data, no design recommendations can be made at this time. However, the design features of this type of joint are such that it should not be used where slab lengths exceed 20 ft.

4. Butt joint

This is essentially a construction joint, and should be treated in the manner of other types of contraction joints requiring load-transfer devices.

(c) *Hinge or warping joints*

There are two types of hinge joints, namely, groove and formed tongue-and-groove. They are used mainly in longitudinal joint construction. At hinge joints, separation of the abutting slabs is prevented by bonded steel tie bars or two-piece connectors. Load transfer is accomplished by aggregate or mechanical interlock, and to some extent by the tie bars.

1. Groove joint

Although the formed groove method is more commonly used, the formation of the groove by cutting the pavement with a concrete saw is rapidly taking its place, and is preferentially recommended.

2. Tongue-and-groove joint

The essential elements of this joint are the formed tongue-and-groove, which provide load transfer, and deformed tie bars to hold slabs in adjacent lanes in intimate contact. The tongue-and-groove may be formed in full-width construction by a preformed metal plate left in place. In lane-at-a-time construction it is recommended that the desired groove in the slab edge be formed by using deformed metal strips or wood strips attached to the forms.

The top edges of the butt type of hinged joint should be finished with a groove so that effective sealing of the joint may be accomplished.

3. Recommended design details for hinge joints are illustrated in Fig. 502(c.).

(d) *Tie bars*

1. The purpose of tie bars is to hold the faces of abutting slabs in intimate contact, thus insuring adequate load transfer. Tie bars are not designed to act as load-transfer devices.

Tie bars are designed to withstand the maximum tensile forces required to overcome the friction between the pavement and the subgrade, from the joint in question to the nearest free edge.

2. Recommended sizes and lengths for different pavement conditions are given in Table 502(d).

3. The practice of using bent tie bars which are later straightened as in lane-at-a-time construction often results in kinked, broken, or misplaced tie bars. To avoid these difficulties it is recommended that such tie bars be made from electric furnace or open hearth billet steel or axle steel, structural grade, or that an approved type of two-piece connector be used. The tensile strength

TABLE 502(d)—RECOMMENDED MAXIMUM SPACING FOR ½- AND ⅝-IN. TIE BARS*

Type and grade of steel	Working stress, psi	Pavement thickness, in.	½-in. diameter bars				⅝-in. diameter bars			
			Over-all length, in.	Spacing, in.†			Over-all length, in.	Spacing, in.†		
				Lane width 10 ft	Lane width 11 ft	Lane width 12 ft		Lane width 10 ft	Lane width 11 ft	Lane width 12 ft
Structural grade billet or axle steel	22,000	6	20	46	42	38	24	48	48	48
		7		39	36	33		48	48	48
		8		34	31	28		48	48	45
		9		30	28	25		48	43	40
		10		27	25	23		43	39	36
Intermediate grade billet or axle steel	27,000	6	24	48	48	47	27	48	48	48
		7		48	44	40		48	48	48
		8		42	38	35		48	48	48
		9		37	34	31		48	48	48
		10		34	31	28		48	48	44
Rail steel or hard grade billet or axle steel	33,000	6	27	48	48	48	33	48	48	48
		7		48	48	48		48	48	48
		8		48	47	43		48	48	48
		9		46	42	38		48	48	48
		10		41	37	34		48	48	48

*Deformations of tie bars conforming to ASTM A305. Working stress based on 0.67 of minimum yield point strength of steel. Over-all length includes 3-in. allowance for centering.
†It is recommended that spacing of tie bars should not exceed 48 in.

of such connectors should at least be that of conventional tie bars which would be required for a given pavement design.

(e) *Load-transfer devices*

1. Recommended minimum practical requirements for dowels for various pavement thicknesses are given in Table 502(e).

2. Recommendations for load transfer at joints have been based on the most commonly used type of load-transfer device, the ordinary round steel dowel. If proprietary load-transfer devices are used in lieu of dowels, they should have a load-transfer capacity equal to or greater than that of the recommended dowel.

TABLE 502(e)—RECOMMENDED MINIMUM DOWEL REQUIREMENTS FOR EXPANSION OR CONTRACTION JOINTS IN HIGHWAY CONSTRUCTION

Pavement thickness, in.	Dowel diameter, in.	Dowel length, in.	Dowel spacing, in.
6	¾	18	12
7	1	18	12
8	1	18	12
9	1¼	18	12
10	1¼	18	12

503—Airport pavement

(a) Joint designs applicable to both highway and runway construction covered in Section 502 are omitted here to avoid unnecessary duplication.

(b) *Expansion joints*

1. Longitudinal

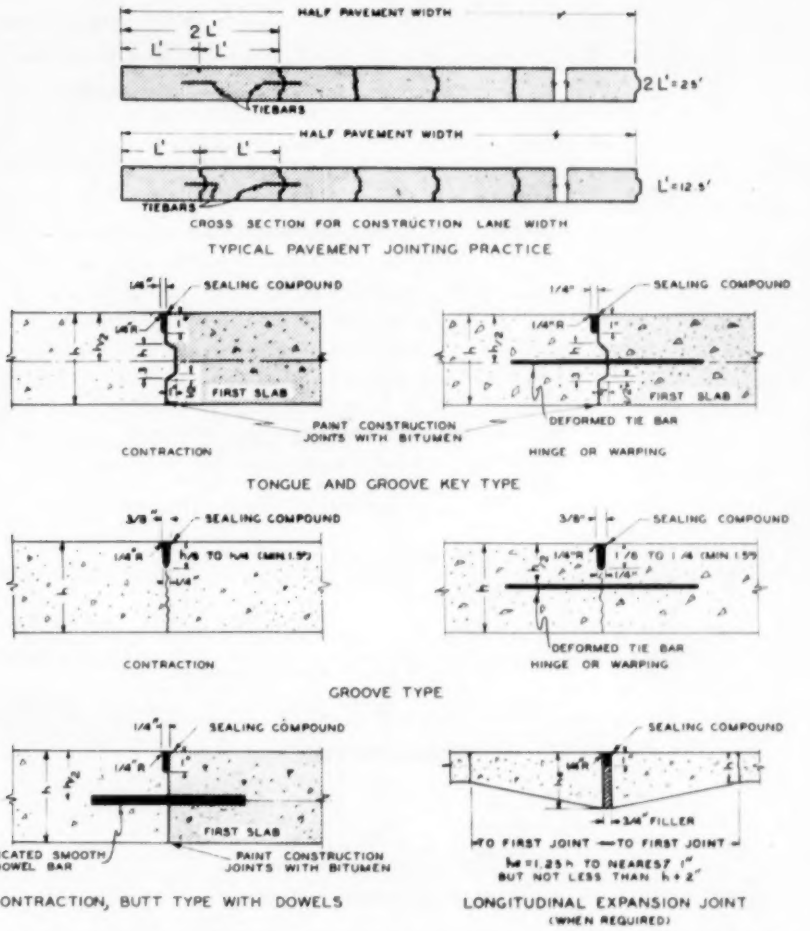


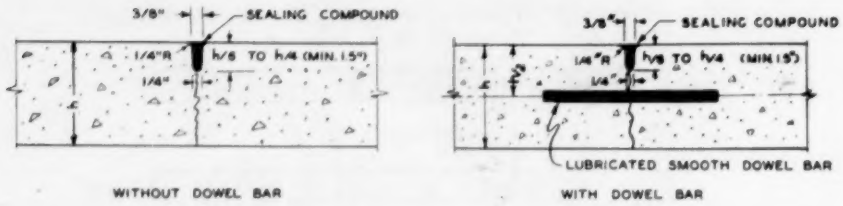
Fig. 503(b)1—Recommended design details for airport runway longitudinal joints

1—Deformed tie bars to be installed in groove and keyed joints occurring within span, 25 ft of free edge of pavement
2—Butt construction joints, with dowels spaced as required, may be used in lieu of keyed joints
3—All joints shall be sealed with approved material

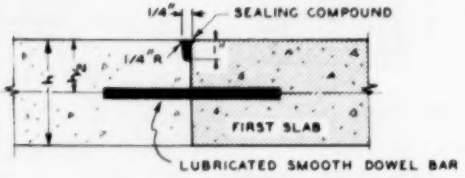
Recommended longitudinal expansion joint design details are shown in Fig. 503(b)1. Dowels should not be used at longitudinal expansion joints, but the edges of the slabs along these joints should be thickened to at least 1.25 times the normal slab thickness. Thickness of the joint filler should be ¾ in.

2. Transverse

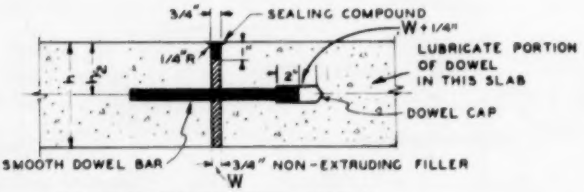
It is recommended that the thickness of joint filler for transverse expansion joints should be not less than ¾ in. nor more than 1 in. [see Fig. 503(b)2].



GROOVE TYPE CONTRACTION JOINTS



CONTRACTION, BUTT TYPE WITH DOWELS



EXPANSION TYPE WITH DOWELS

Fig. 503(b)2—Recommended design details for airport runway transverse joints

1—Dowels shall be used in groove joints in aprons, taxiways, and thickened end sections of runways. They may be omitted in other portions of runways
2—Dowel size and spacing shall be as required
3—All joints shall be sealed with approved material

(c) *Contraction joints*

1. Groove joint

The groove contraction joint is recommended. The groove shall be of sufficient width and depth to perform its function and to permit proper sealing. In general, the depth of groove may range from 1/6 to ¼ the thickness of the slab. Within these limits the depth shall not be less than 1½ in., nor less than 1/6 the thickness of the slab, nor less than the maximum size of aggregate.

2. Keyed and butt joint (construction joints)

These joints naturally result from lane-at-a-time construction, and may

be constructed either with a key (tongue-and-groove interlock), with or without tie bars, or as a plain butt joint with smooth dowel bars. Recommended design details are shown in Fig. 503(b)1.

(d) *Hinge or warping joints*—Depending upon design requirements, hinge or warping joints may be constructed as groove joints, tongue-and-groove joints, or as plain butt joints. In all three cases, tie bars must be employed to hold the abutting slabs in intimate contact. Details for this type of joint construction are shown in Fig. 503(b)1.

(e) *Design of tie bars*

1. The recommended minimum diameter of tie bars for airport work is $\frac{5}{8}$ in. The recommended range of tie bar length is 30 to 36 in. at a spacing of 30 in. on center.

2. The selected size and spacing of tie bars shall be such that the cross-sectional area of steel in square inches, per ft of joint, is at least equal to the amount given by the following formula:

$$A_s = \frac{L' F\, w}{f_s} \qquad (12)$$

in which L' is the distance between the joint in question and the nearest free joint or edge.

3. A tie bar should be long enough to provide sufficient anchorage on each side of the joint to develop the strength of the tie bar. In addition, an allowance should be made for inaccurate centering of the tie bar. A 3-in. allowance is recommended. Expressed as a formula, this length becomes:

$$L_T = \frac{2 f_s\, A_s}{u\, Q} + 3 \text{ in.} \qquad (13)$$

TABLE 503(f)—DOWEL REQUIREMENTS FOR LONGITUDINAL BUTT JOINTS, TRANSVERSE EXPANSION, CONSTRUCTION, AND CONTRACTION JOINTS IN AIRPORT CONSTRUCTION

Pavement thickness, in.	Dowel diameter, in.	Dowel length, in.	Dowel spacing, in.
6	$\frac{3}{4}$	18	12
7	$\frac{3}{4}$	18	12
8	1	18	12
9	1	18	12
10	1	18	12
11	$1\frac{1}{4}$	20	15
12	$1\frac{1}{4}$	20	15
13	$1\frac{1}{4}$	20	15
14	$1\frac{1}{4}$	20	15
15	$1\frac{1}{4}$	20	15
16	$1\frac{1}{2}$	24	15
17	$1\frac{1}{2}$	24	15
18	$1\frac{1}{2}$	24	15
19	$1\frac{1}{2}$	24	15
20	$1\frac{1}{2}$	24	15

(f) *Load-transfer devices*—Some form of mechanical load transfer is recommended at all transverse expansion and construction joints as well as at butt longitudinal construction joints. The recommended size, length, and spacing of dowels for load transfer are shown in Table 503(f).

CHAPTER 6—REINFORCEMENT

601—Scope

This section presents data and recommendations for the most common type of pavement reinforcement. It is limited to reinforcement in pavement slabs relatively free to undergo contraction and expansion throughout their length. Such slabs are not usually over 100 ft long, although data from continuously reinforced pavements indicate that principles of reinforcement for crack control apply to somewhat greater slab lengths, between 100 and 200 ft. Specially oriented reinforcement, such as edge bars, corner bars, and two-way structural reinforcement used in bridge approach slabs, are not within the scope of this report. The effect of the steel in preventing cracks is not considered herein.

602—General principles

(a) *Purposes of reinforcement*—It is intended to insure the following:

1. Intimate contact of adjacent slabs to produce a degree of load transfer across the crack through aggregate interlock so that wheel loads near the crack are supported by the slabs on both sides with correspondingly smaller concrete flexural stress at the crack and so that load deflections will be minimized.

2. Minimization of water seepage to the subgrade soil and the infiltration of soil into the crack.

(b) *Design criteria*

1. When its temperature drops, a pavement slab tends to shorten. This contraction is resisted by the subgrade. The frictional resistance, which increases from the ends to the center of the slab, must be overcome by the tensile resistance of the steel crossing any cracks. The maximum steel stress will occur at a crack at mid-length of the slab, and reinforcement is designed on this basis.

2. Frictional resistance will vary with amount of movement, size of slab, rate of temperature change, and characteristics of the subgrade, reaching a maximum when actual sliding of the slab on the subgrade occurs. For design purposes, a coefficient of friction of 1.5 is recommended. From experience, this value has given satisfactory results, probably compensating for other factors not considered in present design practice.

3. A more refined method for determining the coefficient of friction value when sliding and nonsliding conditions are involved has been developed. This method assumes that the subgrade frictional resistance increases in direct proportion to the square root of the slab movement until free sliding takes place, with a constant coefficient of friction for parts of the slab subject to free sliding. A full explanation of this rational method for selection of

coefficient of friction values will be found in "Design Considerations for Concrete Pavement Reinforcement for Crack Control," by Subcommittee IV, ACI Committee 325, ACI JOURNAL, Oct. 1956, *Proc.* V. 53, p. 341.

4. The cross-sectional area of steel required per foot of slab width is

$$A_s = \frac{F\,L\,w}{24\,f_s} \qquad (14)$$

603—Design requirements

(a) *Minimum size of longitudinal reinforcement*—The size of members is dictated by practical considerations and industry standards. Longitudinal reinforcing members in heavy-duty highway pavement should not be less than No. 3 AS&W gage or approximately ¼-in. diameter. In the case of deformed bar mats, the minimum size member should be that of a #3 bar, or ⅜ in. in diameter.

(b) *Transverse reinforcement*

1. Transverse steel reinforcement should be designed to develop a tensile value comparable to the full strength of the tie bars at longitudinal joints in two-lane or multiple lane-pavements.

2. The minimum size of member should be No. 4 AS&W gage or 0.225 in. in diameter. For bar mats, the minimum size of member should be ¼ in. in diameter.

3. From the standpoint of satisfactory cross-welding, the difference in size between a longitudinal and transverse wire should be not greater than six gage numbers.

(c) *Maximum spacing of reinforcement*

1. For welded wire fabric, the spacing of the longitudinal wires should not exceed 6 in., and of the transverse wires, 12 in.

2. For bar mats the spacing of the longitudinal bars should not exceed 15 in., and of the transverse bars, 30 in.

(d) *Location of reinforcement*—Reinforcement should be placed at a depth not less than 2 in. nor more than ⅓ h from top of slab.

APPENDIX—USE OF DESIGN CHARTS

A401—Highway pavement design

See Fig. A401(a) and A401(b).

A402—Airport pavement design*

(a) *Use of the airport design charts for determining pavement thickness*

1. The proper working stress should be established. This is done by dividing the estimated modulus of rupture of the concrete by the appropriate safety factor.

2. The k value of the subgrade must be determined as explained previously.

3. The type of landing gear, gear load, and tire pressure must be known for the planes which are anticipated on the facility being designed.

*From "Design of Concrete Airport Pavement," Portland Cement Association, Chicago, 1955.

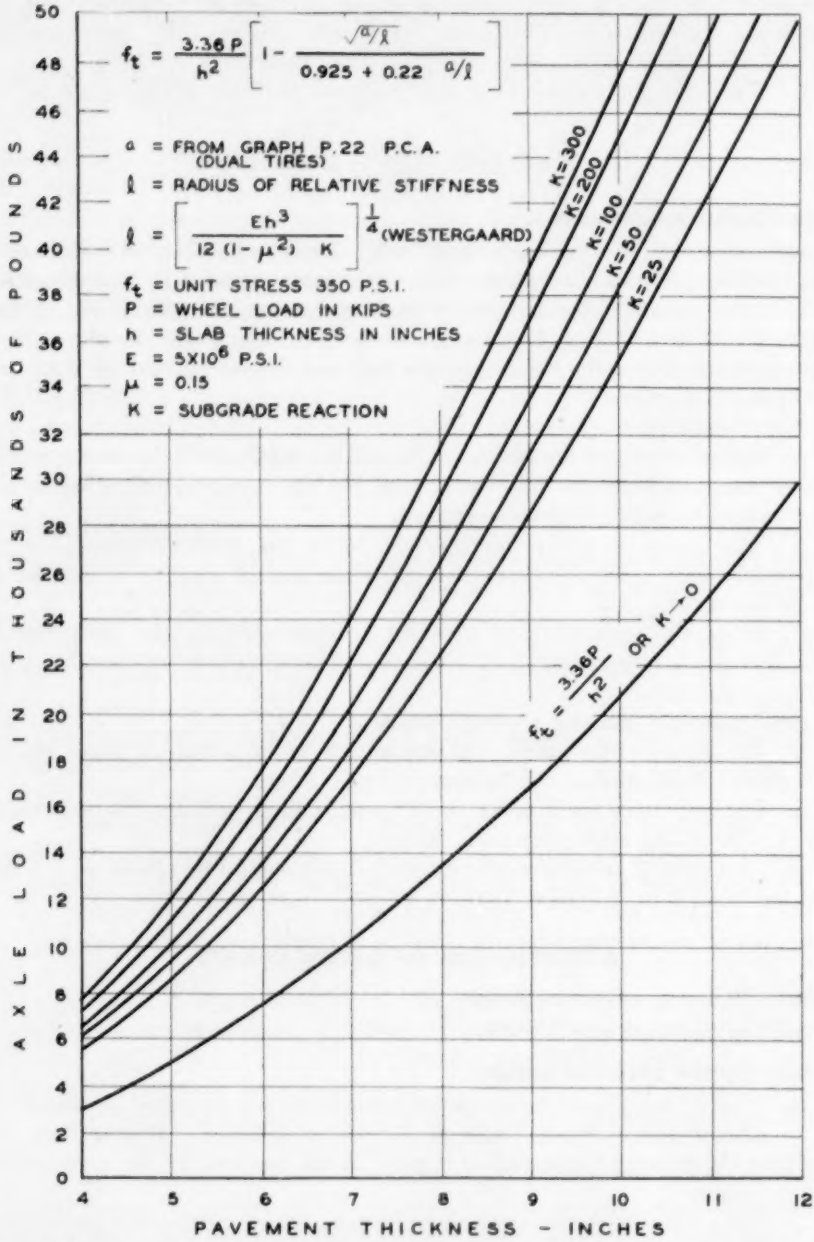

$f_t = \frac{3.36P}{h^2}\left[1 - \frac{\sqrt{a/\ell}}{0.925 + 0.22\ a/\ell}\right]$
a = FROM GRAPH P.22 P.C.A. (DUAL TIRES)
ℓ = RADIUS OF RELATIVE STIFFNESS
$\ell = \left[\frac{Eh^3}{12(1-\mu^2)\ K}\right]^{\frac{1}{4}}$ (WESTERGAARD)
f_t = UNIT STRESS 350 P.S.I.
P = WHEEL LOAD IN KIPS
h = SLAB THICKNESS IN INCHES
E = 5×10^6 P.S.I.
μ = 0.15
K = SUBGRADE REACTION
K=300
K=200
K=100
K=50
K=25
$f_t = \frac{3.36P}{h2}$ OR K→0
AXLE LOAD IN THOUSANDS OF POUNDS
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
4 5 6 7 8 9 10 11 12
PAVEMENT THICKNESS - INCHES

Fig. A401(a)—Design chart for portland cement concrete pavements—protected corners

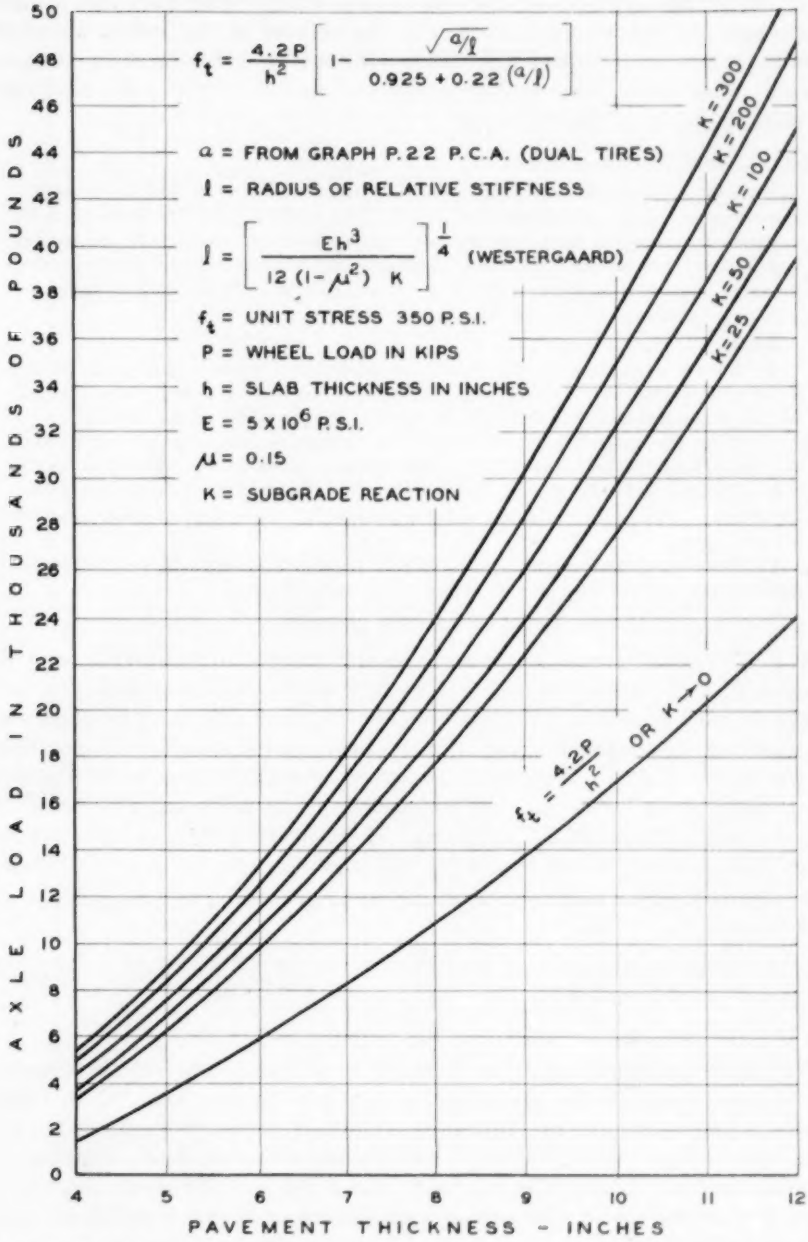

$f_t = \frac{4.2P}{h^2}\left[1 - \frac{\sqrt{a/l}}{0.925 + 0.22\,(a/l)}\right]$
a = FROM GRAPH P.22 P.C.A. (DUAL TIRES)
l = RADIUS OF RELATIVE STIFFNESS
$l = \left[\frac{Eh^3}{12\,(1-\mu^2)\,K}\right]^{\frac{1}{4}}$ (WESTERGAARD)
f_t = UNIT STRESS 350 P.S.I.
P = WHEEL LOAD IN KIPS
h = SLAB THICKNESS IN INCHES
E = 5×10^6 P.S.I.
μ = 0.15
K = SUBGRADE REACTION
K=300
K=200
K=100
K=50
K=25
$f_t = \frac{4.2P}{h^2}$ OR K→0
AXLE LOAD IN THOUSANDS OF POUNDS
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
4 5 6 7 8 9 10 11 12
PAVEMENT THICKNESS - INCHES

Fig. A401(b)—Design chart for portland cement concrete pavements—unprotected corners

4. Enter the appropriate chart at the left margin with allowable working stress and proceed from left to right in the manner of the broken lines with arrows on the charts—read right from allowable working stress to curve for k value, then up or down to design-load line, and from that point read right to slab thickness at the right edge.

(b) *Sample design problem*

Required: The thickness of an apron extension and taxiway for an occasional plane having a dual-wheel load of 80,000 lb. The existing pavement is designed for a 40,000-lb dual-wheel load, and aircraft with loads of this magnitude are making several operations per day. The k of subgrade established from tests on the existing subgrade under the old apron is 200 lb per cu in., and the concrete mix is designed for a 28-day modulus of rupture of 650 psi.

Solution:

1. Allowable working stress equals the modulus of rupture, increased to the 6-month strength (650 × 114 percent = 741 psi) divided by safety factor (say 1.7 for the occasional 80,000-lb dual) or 741 divided by 1.7, which equals 436 psi.
2. Enter the design chart, Fig. A402(a) for dual-wheel loads, on the left margin at a stress of 436 psi and proceed right horizontally to the desired curved line for k 200.
3. Proceed vertically to the diagonal line for the dual-wheel load in question (80,000 lb). The heavy solid lines indicate average tire pressures commonly used for the load given.
4. Proceed right horizontally to the edge of the chart and read the required thickness, in this example, 10.8 in. Probably 11 in. would be used.

(c) *Use of charts for review of existing pavement*

(1) The design charts may also be used in reverse to determine the stress caused by a given load in a pavement of a known thickness and on a subgrade whose k is known. For example, use of the charts could be illustrated by checking the adequacy of the 11 in. obtained above for unlimited operations of the 40,000-lb dual. Starting at 11 and moving left across to the 40,000-lb load, down to the 200 k line, and horizontally to the left edge, a stress of 238 psi or a safety factor of 2.8 at 28 days is obtained. This indicates that the pavement is more than adequate for this loading.

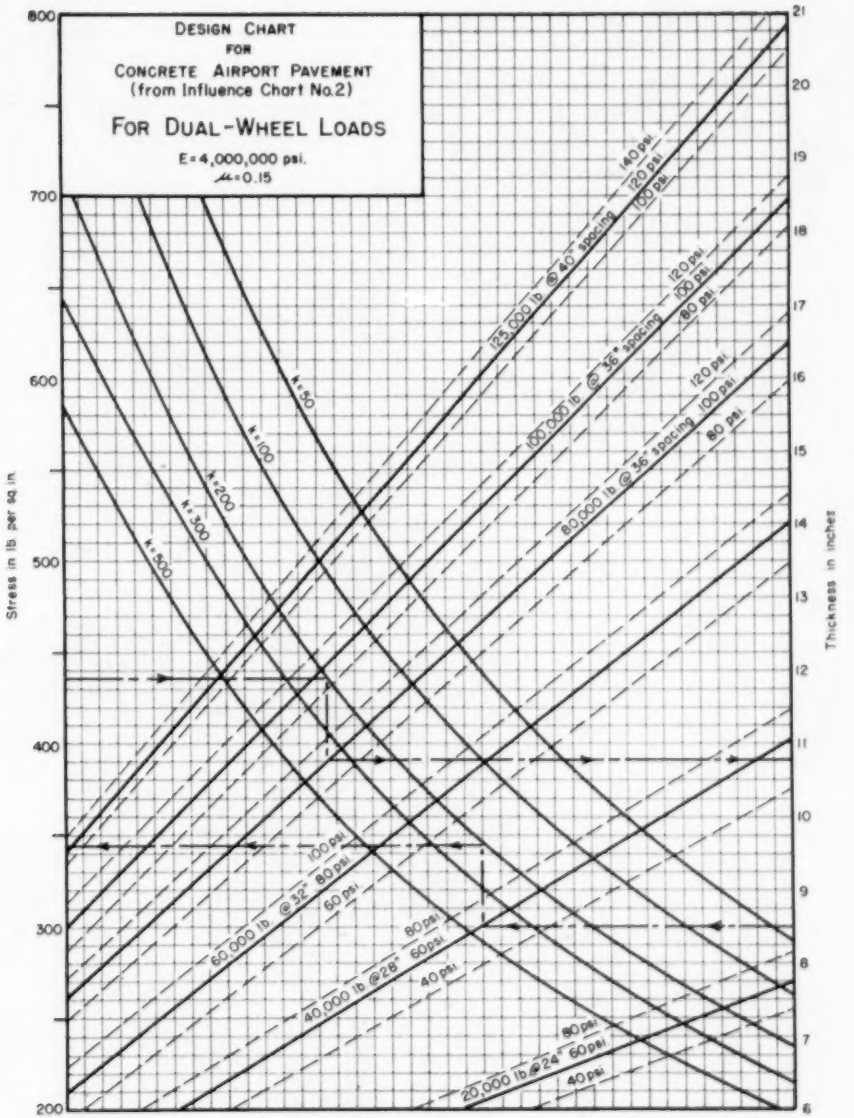
Loads or k values other than those shown can be used by interpolation on the charts.

The same procedure would be followed on the other design charts, Fig. A402(b), (c), (d), and (e).

(d) *Tire pressure*

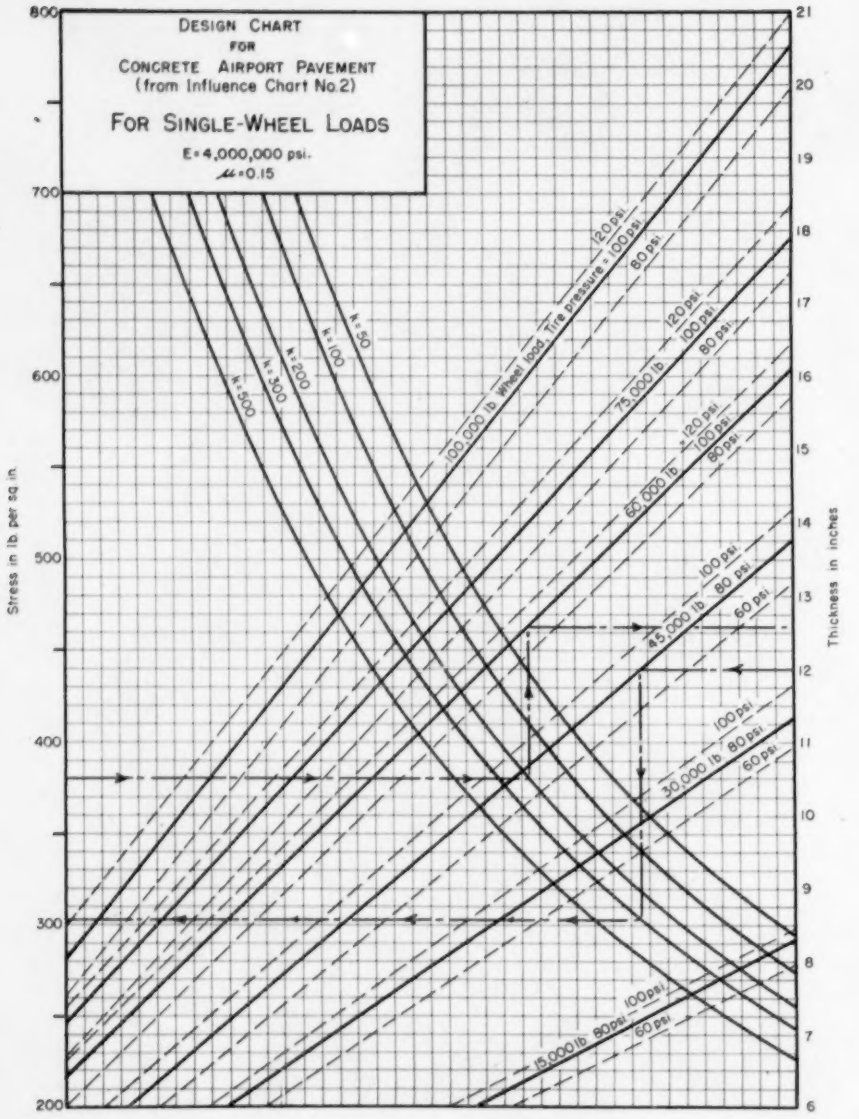
The dashed diagonal lines are included to permit the use of these charts for a wide range of tire pressures for any landing-gear load. The heavy diagonal lines indicate tire pressures commonly used for each load. For example, in Fig. A402(a) for single-wheel loads with a load of 75,000 lb, the average pressure indicated by the heavy line is 100 psi. However, the dashed lines permit the use of tire pressures of 120 psi or 80 psi if either of these values should be desired. Intermediate values may be interpolated.

Fig. A402(d) and A402(e) were prepared for military jet aircraft and include the range of tire pressures common for these planes.



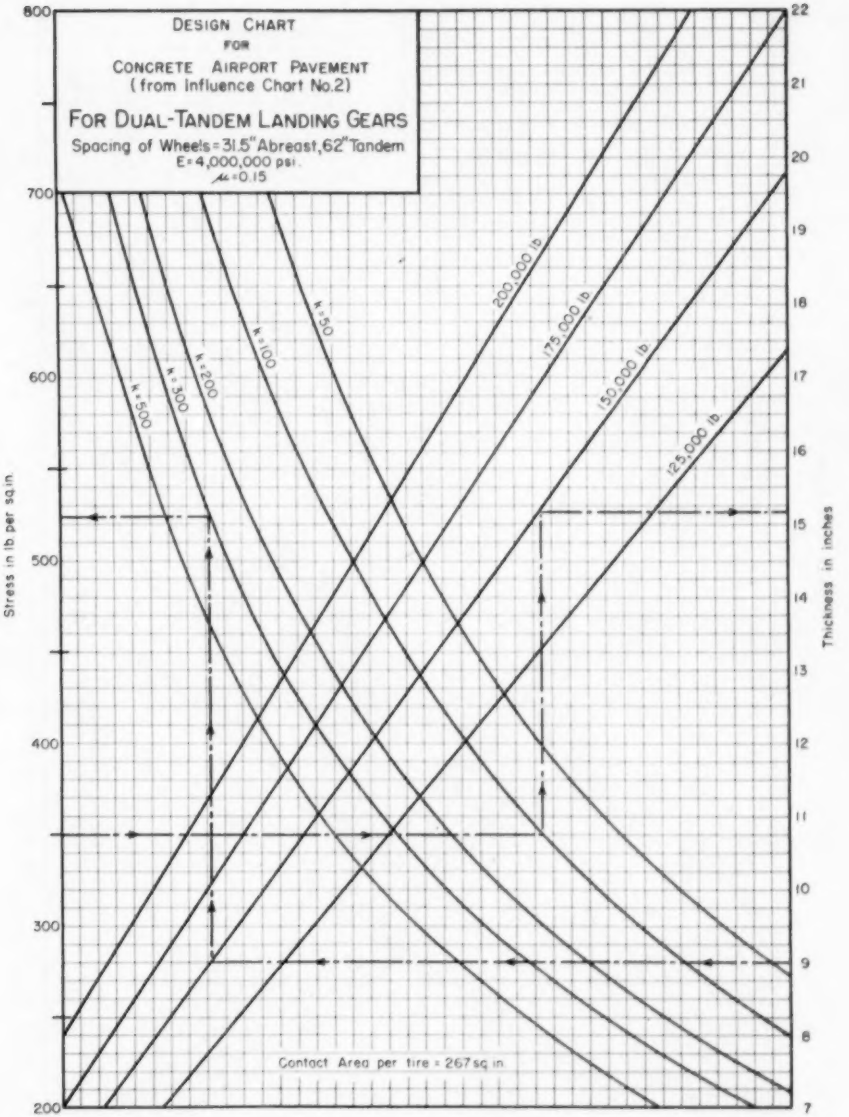
Courtesy Portland Cement Association

Fig. A402(a)



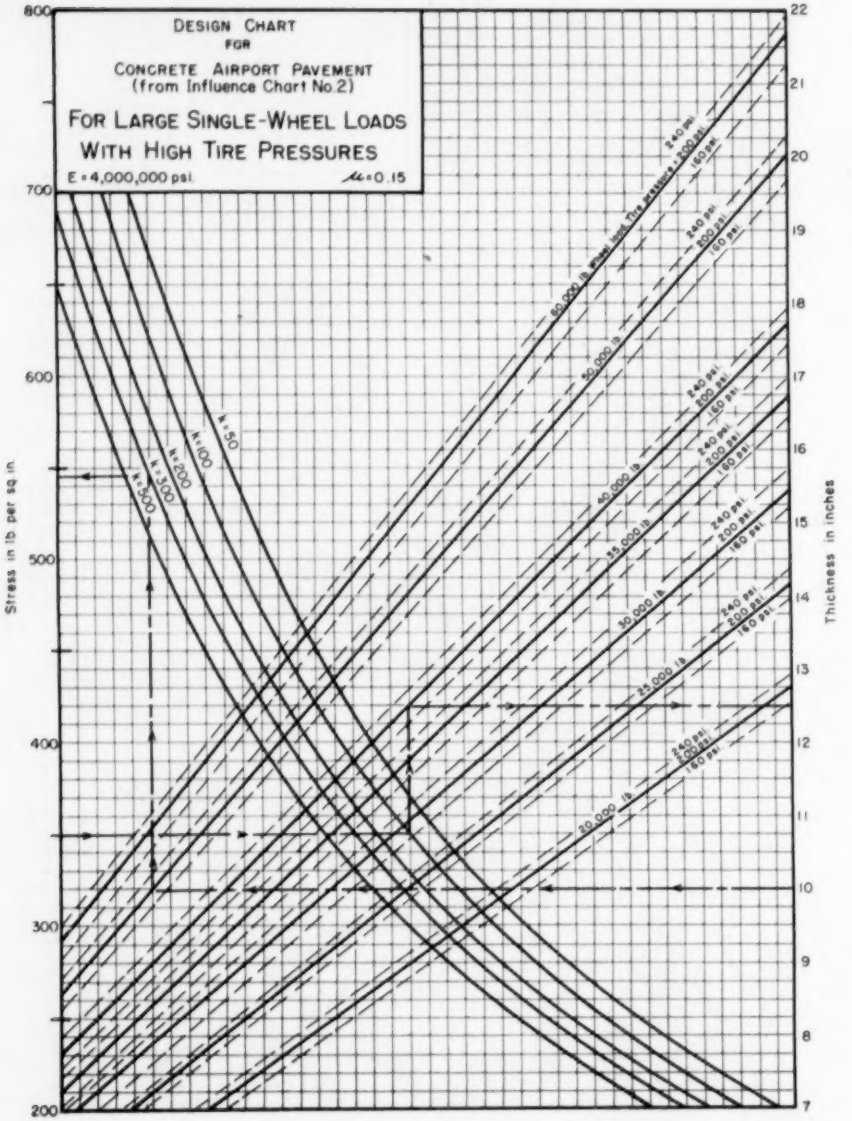
Courtesy Portland Cement Association

Fig. A402(b)



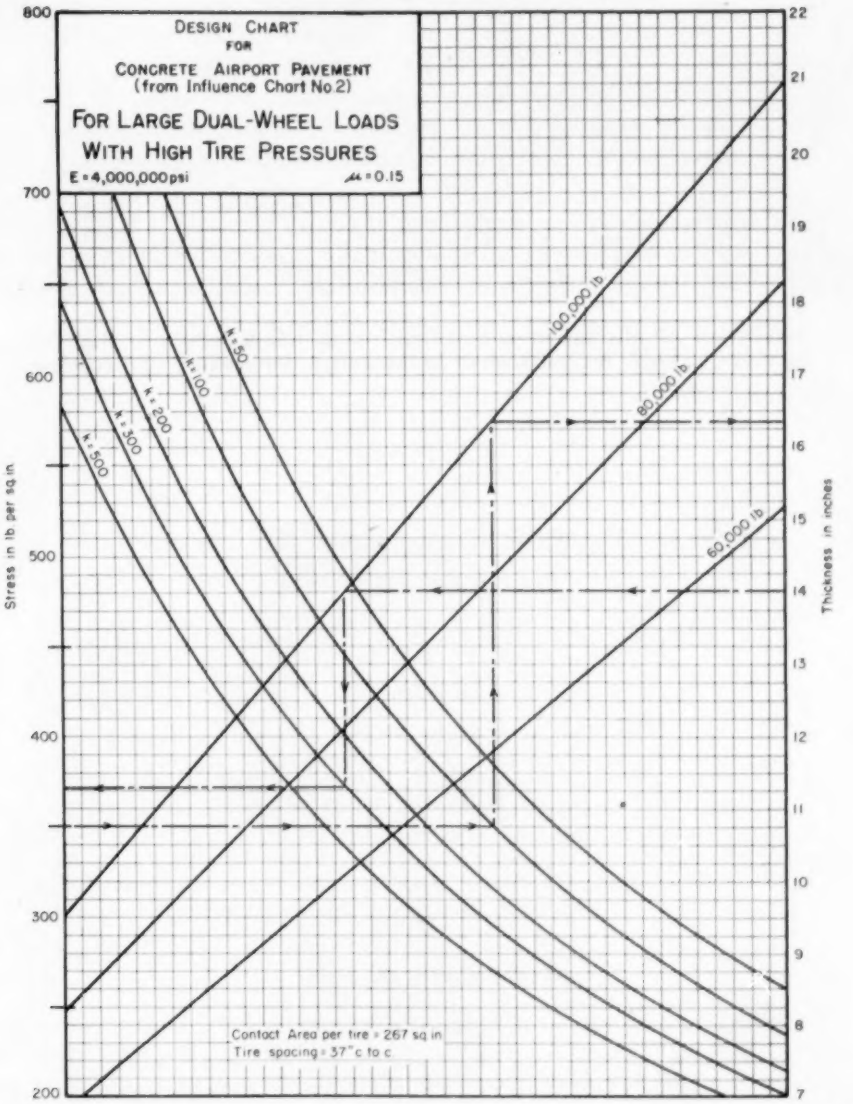
Courtesy Portland Cement Association

Fig. A402(c)



Courtesy Portland Cement Association

Fig. A402(d)



Courtesy Portland Cement Association

Fig. A402(e)

(e) *Other landing gear*

Where the thickness is required for a dual-wheel landing gear with a specific spacing other than those shown, the required thickness may be obtained from Fig. A402(a) and A402(e) by applying the following approximate correction factors:

1. For increases in center-to-center tire spacing up to 10 in., the required thickness should be reduced by 0.6 percent for each inch increase.

2. For each inch decrease in dual spacing up to 10 in., the required thickness should be increased by 0.6 percent.

Similar correction factors cannot be applied for dual-tandem landing gears other than that given in Fig. A402(c). If the spacings are changed appreciably, a special design chart can be developed.

Discussion of this report should reach ACI headquarters in triplicate by June 1, 1957, for publication in the Part 2, December 1957 JOURNAL.

Fig. 1—Artist's rendering of Litchfield County Hospital, Winsted, Conn.

Title No. 53-40

Six Stories of Prestressed Slabs Erected by Lift-Slab Method*

By JAMES S. MINGES† and DONALD S. WILD‡

SYNOPSIS

Describes construction of a five-story hospital with solarium. Lift slabs are 43 ft x 182 ft x 8½ in. for the first five floors, and 41 ft x 80 ft x 8½ in. for the solarium roof which was lifted 62 ft. Special shear blocks made it possible to hold the slab in its lifted position with 1¾ in. pins during welding. This freed lifting equipment to proceed with the next slab. Columns were 14-in. wide flange sections erected in two parts. Top slabs were lifted to the top of the first section and held temporarily while bottom floors were set in final position. While bottom floors were being welded, the top section of the columns was erected and the top slabs placed in final position.

GENERAL DESCRIPTION

Winsted, Conn., one of the cities hardest hit by the memorable floods of August, 1955, will have for use soon one of the most modern hospital facilities in New England, the new addition to the Litchfield County Hospital (Fig. 1).

*Received by the Institute Oct. 17, 1956. Based on a paper presented at the ACI 52nd annual convention, Philadelphia, Pa., Feb. 22, 1956. Title No. 53-40 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 8, Feb. 1957. *Proceedings* V. 53. Separate prints are available at 50 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than June 1, 1957. Address P. O. Box 4754, Redford Station, Detroit 19, Mich.

†Member American Concrete Institute, Partner, Marchant & Minges, West Hartford, Conn.

‡Member American Concrete Institute, Engineer in Charge, Structural Department, Marchant & Minges, West Hartford, Conn.

The new 80-bed building will adjoin and eventually replace the existing 50-year-old hospital structure.

The new addition, believed to be the tallest prestressed structure ever erected by the lift-slab method, has five full stories topped by a solarium roof slab; total cost is anticipated to be $1,100,000.

The five full floor and roof slabs, which have been lifted at this writing, are 46 ft wide and 183 ft long, 8½ in. thick, and of the flat plate type of construction. The uppermost sixth slab, the solarium roof, is 41 x 80 ft, 8½ in. thick, and has been lifted a total of 62 ft to its final position. All six slabs were cast one on top of the other, on the basement slab on grade. Four slabs have a 3-ft overhang for sun control on the south.

Side walls carried by the slabs are of glass and light metal panels to minimize dead load. End walls are of masonry built around ends of slabs after the slabs were hoisted into position. Total height of the building to the top of the penthouse structure is 78 ft 4 in.

The entire structure is supported by only fourteen 14-in. wide flange columns spaced 26 ft on center each way with each slab cantilevering on all four sides during erection. Having just 14 columns, the construction is particularly adapted to the standard lift-slab console equipment. Each slab can be lifted as a whole, thus eliminating cast-in-place connecting strips. Typical slabs are approximately 8000 sq ft in area and weigh 400 tons each. Columns will be wrapped and fireproofed with 1 in. of perlite plaster. Column collars have been recessed into slabs to allow metal lath and plaster to be applied at the ceiling where concrete is exposed in order to provide sufficient fireproofing cover. All spaces having exposed concrete will be painted with a cement-base paint.

STRUCTURAL DESIGN

General problem

The structural analysis of the prestressed concrete slabs consisted of the design of four essentially identical slabs (first, second, third floors, and the main roof), a slightly different ground floor slab, and a completely different solarium roof slab. The ground floor slab differed in that it lacked the 3-ft cantilevered shade along the south side as well as one stairwell opening. The ground floor slab had almost tripled normal live loads on the east end bay to accommodate x-ray equipment. This slab was also subjected to the upkick from a curved independent 11-ft cantilever exterior service canopy slab, which can be seen at the end wall of the building in Fig. 1. The solarium roof slab differs from other slabs mainly in size. About half the size of the floor slabs, it is supported on only six of the 14 columns. So great was the loss of continuity and stiffness in the solarium roof because of openings for elevators, stairs, and mechanical equipment, that an upturned monolithic beam 5 ft 6 in. wide and 16 in. deep was needed, spanning transversely between the two westerly columns.*

*This beam is visible at the center of Fig. 8.

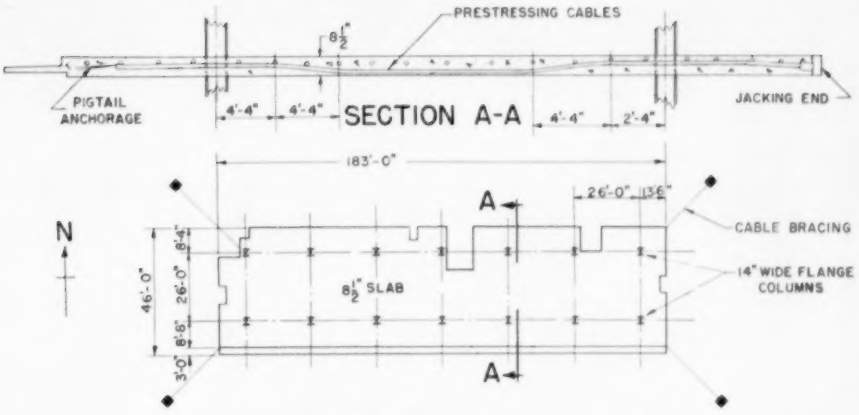


Fig. 2—Typical floor plan and slab cross section

The typical full sized slabs are 46 ft wide (including the 3-ft shade along the south) and 183 ft long, supported on two rows of seven columns as shown in Fig. 2. Column rows are 26 ft apart with columns in each row evenly spaced at 26 ft. These 8½ in. thick slabs cantilever 13 ft 6 in. beyond each end pair of columns in the longitudinal direction, 11 ft 8 in. out from the south column row, and 8 ft 4 in. out from the north column row.

Column rows were spaced as practically as architecturally possible in the transverse direction to provide essentially equal maximum negative and positive moments.

Three architecturally necessary openings (two stairwells 8 x 11 ft, and an elevator shaft 9 x 17 ft), seriously complicated the design by occurring in regions of critical stress. In fact, completion of the analysis and design clearly indicated the advisability and practicality of locating such stairwells and elevator shafts outside the main slab areas on future buildings of this nature.

Slab design

Actually, two completely different slab designs were made. The first resulted in a 7⅝-in. slab with 5000 psi concrete and considerable prestressing reinforcement, while the second and final design utilized an 8½-in. slab with 4500 psi concrete and approximately half of the previous prestressing reinforcement. Several factors contributed to this sizable reduction and were necessitated by the urgency to cut the total building cost when the job was rebid. Major reducing factors were as follows:

1. Changing end walls from porcelain panels to masonry enabled them to act as wind shear walls, eliminating relatively large wind moments from the slabs.
2. Masonry end walls were also utilized for live load bearing of the 13 ft 6 in. cantilevered ends (but not for dead load, since self-support was necessitated by the lift operation).
3. Changing partition construction from concrete block to hollow gypsum block reduced partition dead load from 45 psf to 27 psf.

4. Instead of the original ultraconservative full load distribution on both transverse and longitudinal bents, a more logical triangular load distribution pattern, in conjunction with perimeter and other concentrated loads, was utilized.

5. Increasing the slab thickness ⅞ in. afforded an average 25 percent increase in eccentricity of the prestressing cables from the slab neutral axis.

Bent analysis

Bent analysis was handled in the usual manner by moment distribution, employing the bay width of 26 ft for the slab and two columns in the transverse direction. Longitudinal bents were composed of a slab width equal to half the building width with seven columns for support. Absolute stiffnesses of concrete slab and steel were necessitated, with the modulus of elasticity of concrete taken at 80 percent of 1000 f_c' to account for plastic flow.

Dead load moments were calculated for the slabs with purely simple supports, since the columns could offer no restraint during lift operations. Live load moments, however, were calculated with assumed full rotational restraint for slab-to-column connections. Restraint results from welding bottoms of cast lift collars to columns and welding of special steel wedges which were inserted between columns and collar tops. Also, the masonry end walls were utilized to support the 13 ft 6 in. cantilever ends for live load only.

Prestressing design problems

The resultant maximum moments, both positive and negative, dictated the relative positioning of transverse and longitudinal cables so that the displacements of each, top and bottom, from the neutral axis of the slab resulted in maximum efficiency. Thus, transverse cables were outermost at

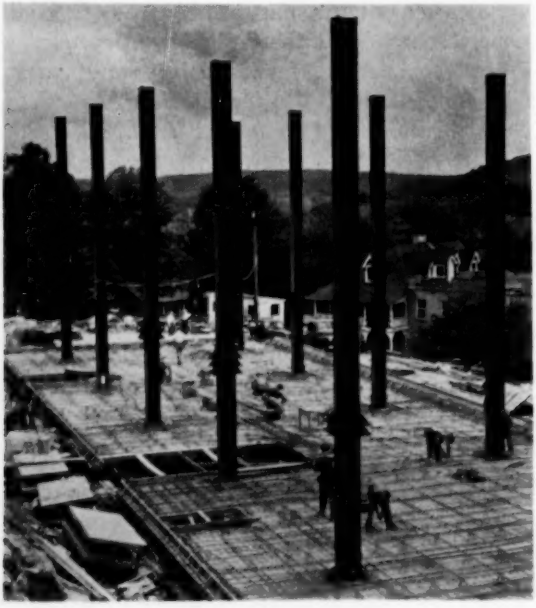


Fig. 3—Cable and reinforcement placement

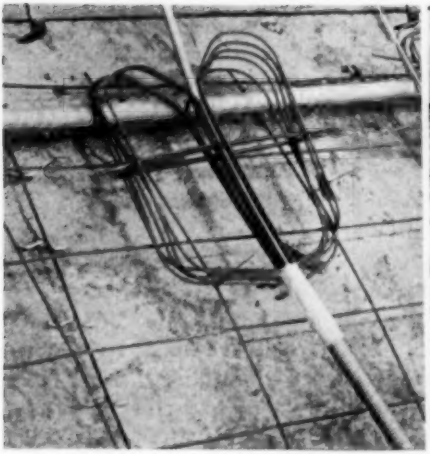


Fig. 4—Pigtail anchors of prestressing unit

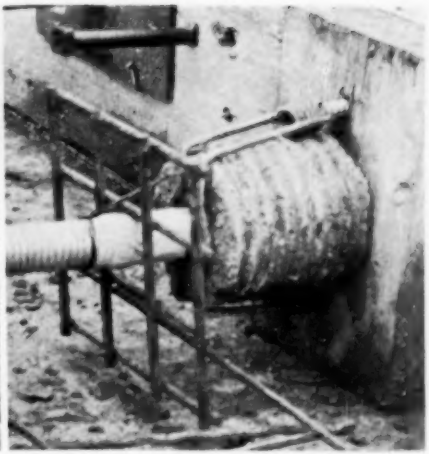


Fig. 5—Cone assembly of prestressing unit

bottom of slab, and innermost at top of slab, with 1 in. clear cover over cables at both faces. Cable arrangement can be seen in Fig. 3.

Relatively stiff flat sheets of 612-66 electrically welded mesh on low bolsters were used at bottom of the slabs to support lowest draping transverse cables, enabling them to be wired down adequately, while also providing otherwise lacking temperature reinforcement prior to stressing. Some extra, mild steel was necessary at columns.

Design was based on ten-wire prestressing units of the Freyssinet system, which were eventually used from the resultant low bid, although specifications encouraged competition from other types. Long longitudinal cables were designed for post-tensioning from both ends, while shorter longitudinal and transverse cables were stressed from only one end, employing a "pigtail" anchorage at dead ends and a cone assembly at the jacking ends (Fig. 4, 5).

A 7-in. diameter was used for "pigtail" bends, with sharp end return. Half of the ten wires were fanned vertically each side of the grouting extension tube. Friction factors for the ten 0.196 in. diameter, special, straight stress-relieved uncoated wires in 1 in. inside diameter flexible metal hose were taken at $K = 1/1000$ and $f = 0.2$ according to criteria of the Bureau of Public Roads and Freyssinet company recommendations.

Together with these losses and the usual 15 percent loss allowance from initial prestress for shrinkage and creep, was the further loss in tension, created by the added friction of cable undulations. Cable draping from top of slab at negative moment regions to bottom of slab in regions of positive moment involved a reversed curve which was made within a length of 4 ft 4 in. Curvature was assumed parabolic, and the complete curve, from top to bottom of slab, amounted to a total angular change of approximately 19 deg 40 min.

Thus, total friction became relatively high for typical longitudinal cables, reducing jacking end stress from 161,000 psi to only 95,400 psi at midlength. This necessitated checking moments with respective prestress forces for all bays, since the more obvious larger moments at end bays were not necessarily indicative of the most critical stress regions.

Progressing from negative moment regions at columns, to regions of positive moment, transition drapes from top to bottom of slab were started 4 ft 4 in. out from column center lines, or one-sixth of the span between columns. This was established, tentatively, by an educated guess, for all cables in both directions, then later checked and relocated at some columns for greater efficiency. Exact transition point was found to be not critical since relatively low tensile stresses in these inflection regions were easily taken care of by the compressive stresses created by transitioning cables. No tension results at either surface of the slab from the prestress force for a distance of 2½ ft where transitioning cables lie wholly within the middle third of slab thickness.

The total 26-ft transverse bent width of slab was divided into column and middle strips, each 13 ft wide (typical for any flat plate analysis). The longitudinal bent width was also divided similarly, but with all of north and south side cantilevers considered as extended column strip width, leaving a middle strip width of only 6.5 ft for each long bent.

In the transverse direction 12 ten-wire cables were symmetrically located within a typical 26-ft bay, the result of a quick preliminary trial and the desire to space cables no further apart than about 2½ ft. The first cable was placed 1 ft from the column center line, the second cable 1 ft from the first, the third cable 2 ft 3 in. from the second, and the rest at 2.5 ft, reducing to the same closer spacing again at the next column line. The closer spacing at the columns was intended to confine more prestress force within the more critical column strips. It was also necessary to determine the proper pattern for spread and overlap of prestress forces from the individual cables near their ends, where complete overlapping of cable forces was not justified as it was farther away from cable ends. The assumption used was that the force from the four cables at 1-ft spacing, at center of column strips, was distributed along a 45 deg line, from ends of outer cables in this group, toward the middle strip. Similar distribution was assumed for the remaining eight cables of a bent width. This grouping was found to be helpful in determining the proportion of total prestress force to be allotted to column and middle strips at any location.

Strength and number of cables in transverse direction were picked to just satisfy maximum positive moment, and maximum negative moment near north side column line, both with maximum efficiency. The higher negative moment near south side column line was then satisfied with the addition of eleven #6 mild steel reinforcing bars 13 ft long, spaced relatively closely between prestress cables, at top of slab in center of column strip. Plans called for three of these rods to be cut in half and fully butt welded to cast steel lifting collars.

These three bars were intended not only to unite the collar solidly with the slab for tension continuity, but also to help fill the 2-ft gap between the two center cables in a region of high tensile stress. However, the quality of butt welds in the field was so unsatisfactory that it was decided to substitute two #8 bars in place of these three #6, placing one #8 on each side of the column, immediately adjacent to the lifting collar.

Strength and number of longitudinal cables were selected for maximum positive moments, and mild steel bars added as needed for negative moments near columns. A total of 20 longitudinal cables were used, with placement similar to those in the transverse direction.

Design stresses

Because of the relatively large 30 in. square lift collars, maximum negative moment at column center lines was reduced 15 percent at the critical design section. Punching shear, which could have been a definite problem at the supporting collars with a non-prestressed slab, was of no real concern here, since the principal tensile stress amounted to only 60 psi at a typical interior column where helpful compressive stresses were lowest due to longitudinal friction losses.

Although a tension stress of 0.04 f_c', or 180 psi in this case, is generally allowed in a completed structure of prestressed concrete without proper reinforcement, it was deemed advisable to eliminate any such tension, the intent being to render the slab more capable of taking unpredicted localized tension that might result from unknown factors involved with the application of present day flat plate theory and analysis. To accomplish this, stresses from prestress, dead load, and live load were investigated at top and bottom surfaces of the slab, in both column and middle strips, at 3-ft intervals, throughout the length and breadth of the structure. Distribution of bent moments to column and middle strips, at regions between columns and midspan, was accomplished by interpolating between the ACI Code 76 percent–24 percent distribution at columns and the 60 percent–40 percent distribution at midspan. Resultant stresses from prestress, dead, and live loads were than tabulated and combined.

This tabulation of stresses was simplified in the transverse direction, since the slab, being supported by only two columns for a bent, was determinate with no secondary stresses resultant from the undulating prestress forces. The longitudinal direction, however, was complicated by being indeterminate, making secondary stresses from undulating prestress forces a definite concern.

Secondary stresses were evaluated after first finding the support reactions resultant from the prestress forces, using the conjugate beam method for finding deflections. The resultant magnitude of these secondary stresses was such as to increase the primary cable stresses an average 20 percent at interior columns, and to reduce the primary cable stresses an average 26 percent at midbay areas.

The completed tabulations indicated that unreinforced tensile stresses in the transverse bent existed at only one area, about 4 ft south of the north side columns. These tensile stresses amounted to only 71 psi and 51 psi at bottom of slab for the column and middle strips, respectively. A 2-ft northward shift in the downward transition point for cables eliminated this tension. This can be seen in Fig. 2, section *A-A*. The resultant unsymmetrical location of cable transition points, within the transverse 26-ft span, was due to the northward shift of the northern inflection point caused by the larger south side cantilever. A similar adjustment was made near extreme ends of longitudinal cables, where stress tabulations indicated magnitude of tensile stresses at bottom of slab, for completed structure, to be 169 psi and 135 psi for column and middle strips, respectively. This was caused by the original upward drape of cables reaching top of slab only 4 ft 4 in. from end of slab, so that cable stresses were considerably greater than cantilever stresses at this region during the lift condition. Here, also, the final load condition, with end of slab on bearing wall, created additional tension at the bottom of the formerly cantilevered slab, so that an inward shift of about 7 ft for this transition point was required to eliminate all tension.

Actually, compressive stresses remain relatively low throughout the slab, nowhere exceeding 500 psi, but the 4500-psi concrete was deemed advisable for a project of this nature. It affords better resistance for possible unknown, unreinforced tensions, as well as making possible earlier tensioning of cables, and lifting of slabs. Plastiment was added to the concrete to reduce the amount of water and shrinkage and to improve workability.

Deflections

A theoretical check on deflections indicated that north and south edges should deflect upward about $\frac{5}{8}$ in. during lift. This deflection is due almost entirely to the slab dead load, with upward direction attributed to the large 26 ft interior span. Prestress forces contributed only about 7 percent. Computations indicated that an additional upward deflection of $\frac{1}{4}$ in. could be expected after application of live loads. Live load deflection of longer east and west end cantilevers will be nonexistent because of end bearing walls. Deflection of these ends during lifting was calculated to be negligible.

Actual deflections in the transverse direction were checked during lifting and found to be considerably smaller than calculated, and slightly in the reverse direction—about $\frac{1}{8}$ in. downward at edges and about $\frac{1}{8}$ in. upward at center of 26-ft span. Inconsistencies in screeding and troweling slabs level made accurate checking virtually impossible. Extra stiffness could well have been due to the actual greater concrete strength (between 4500 and 6500 psi at 28 days) than was used in calculations.

PRESTRESSING OPERATION

Makeup of prestressing cables was hampered slightly by the rugged terrain, which necessitated placement of makeup table on top of the hill immediately

Fig. 6—Prestressing unit assembly table

north of the structure (Fig. 6). Here the ten wires for each cable were machine fed, singly, into the sheaths which were precut to required lengths. Completed cables were then carried down the hill and placed. Proper placing and draping of the cables was easily accomplished by inexperienced laborers who were guided by a three-dimensional scale model constructed by the contractor, as shown in Fig. 7. Vertical scale on the model was greatly exaggerated to clarify transition points and proper chairing elevations.

A minute change in one transition point was made in the field to enable simple placement of transverse and longitudinal cables in alternate groups without the necessity of snaking them through one another. After some slight shifting of cables to avoid interference with plumbing and heating sleeves and electrical boxes, they were wired to each other at intersections, to supporting chairs, and to the rigid mesh, to resist movement during concreting.

Stressing was accomplished by inexperienced laborers and one steel worker also new at prestressing, after one day of instruction by a Freyssinet field

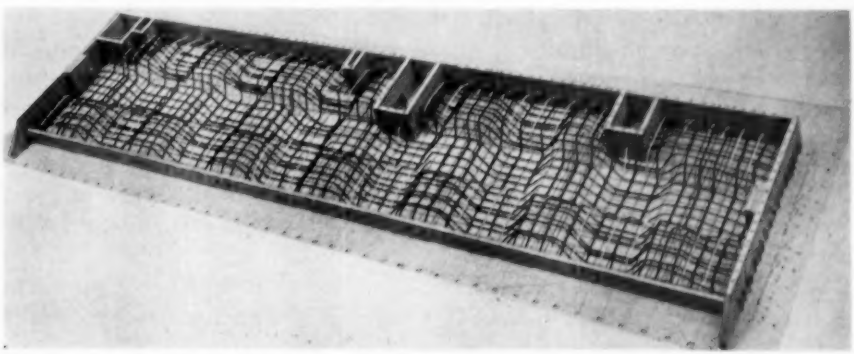


Fig. 7—Contractor's model showing layout of prestressing units

Fig. 8—Prestressing operation. Note upturned monolithic beam of solarium roof slab near center of picture

engineer. In general, two men handled the one jack used for transverse cables, with the steel worker operating the hydraulic pressure controls and recording both actual pressures and elongations. For longitudinal cables which were stressed from both ends, in addition to the two jacking teams, another man was placed at the farther jack to operate an adjustable sign which conveyed to the control operator the exact elongation (to the nearest 1/16 in.) there at any moment, so that coordination with proper pressures was almost instantaneous. In Fig. 8 the control operator and the hydraulic apparatus can be seen in the background, as well as the back of the elongation sign, and a man at the jack in the elevator shaft cut-out. It is particularly noteworthy that such an inexperienced crew did not break any wires internally, although over 6000 individual wires were stressed.

In general, pressures ran slightly higher than those calculated to be consistent with required elongations, possibly because friction was greater than assumed. In view of this, it was decided to use a water-soluble oil flushed through longitudinal cables just prior to stressing, exercising care not to let oil remain on anchoring ends. Deformation of anchorage cones, after stressing and plugging was found to average about $\frac{1}{4}$ in.

After stressing and plugging anchorage cones, a grout of two parts portland cement to one part water by volume, having the consistency of thick paint, was forced through all sheaths to form a permanent protection for the stressed wires. Procedure here was to first flush a sheath with water until a steady flow came out the far end of the grout extension tube. Then the grout mixture would be immediately forced through. When a steady flow of grout was attained, the tube was plugged and pressure maintained briefly before plugging the injection end.

Before actual construction of the slabs, doubt as to whether grout could be forced throughout the sinuous 183-ft length of longitudinal cable prompted installation of midlength grout extension tubes. These were to enable grouting from midlength of slab to each end in the event that grouting from one end

should be made difficult by too much friction. Since no problem was met in grouting from one end, midlength extension tubes were merely plugged.

Wire extensions were then burned off and bent after grouting, as can be seen in Fig. 8 at slab edges. The recess in the edge of slabs, for anchorages, was grouted in later.

ELECTRICAL CONDUITS

Insofar as the trades are concerned in hospital construction, the electrical contractor's work is most affected by the lift-slab operation. Many conduits for the various electrical systems must be buried in the slabs. On this project there is one centrally allotted place on each floor where the conduits for all electrical systems must originate. This means that on most floors there are about 70 conduits which have to terminate in one area. Instead of stubbing up 70 conduits, the problem was solved by employing a heavy gauge vertical plate which is anchored to the slab and will act as the slab edge form at that location, as indicated in Fig. 9.

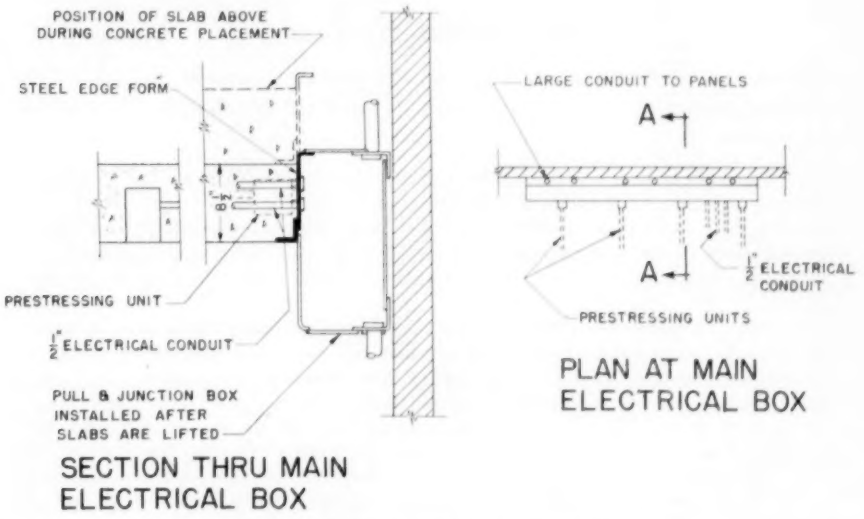


Fig. 9—Electrical pull and junction box

All conduits were fastened to this plate (through knockout holes) by means of lock washers. After the slabs were secured in their final position, a metal wiring trough was fastened to this plate at the floor and the wire from the various electrical systems will be brought up to the panels, several at a time, through large pipe conduits. Steel edge forms at junction box can also be seen in Fig. 10. Conduits in the slabs terminate either in a deep box or are bent so that the ends are provided with a coupling and metal plug at the surface of the slab. The electrical contractor chose to stub his conduits in

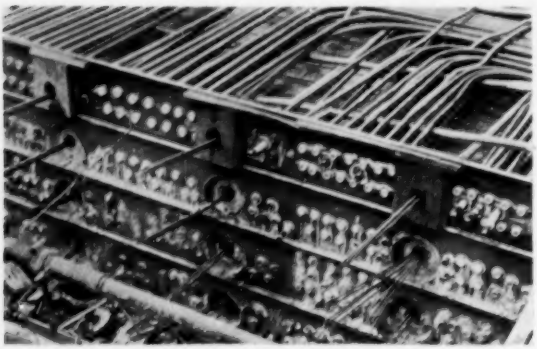


Fig. 10—Conduit at pull and junction box location

tin cans at partitions so that he could later bend the ends of the conduit in order to take care of future adjustments.

Insofar as the heating, air conditioning, and plumbing piping and ductwork are concerned, chases were formed in the slabs and sleeves of sheet metal employed where necessary.

PREPARATION OF SLABS

During the preparation of the slabs for placing concrete and steel reinforcement particular attention was paid to the sequence of operations as outlined below:

1. Foundation work was constructed to underside of basement floor slab. Footings were all founded on bed rock.
2. After backfill under basement slab was placed and fully compacted, the steel columns were erected and plumbed with leveling bolts. Lift collar castings were then strung on the columns and temporarily pinned to columns a few feet above basement slab.
3. After all mechanical piping, electrical conduits, and steel mesh reinforcing were placed, the basement slab on grade was cast and depressions for quarry tile and other finishes were made in the slab. Depressions were filled with dry sand to within ½ in. of the surface and a skim coat of concrete poured over the sand to provide a form for the slab above. The skim coat was removed later by the contractor. The six prestressed, lifted slabs have no depressed areas for finish as does the basement slab.
4. Concrete was then given a steel trowel finish and kept moist for about 18 hr at which time two coats of a bond breaking medium were applied with a simple back-tank sprayer. This material prevented sticking and bonding of the lifted slab to the basement floor slab; it also acted as a curing compound for the slab. The material used was Techkote #400-YS, which dries sufficiently in 30 min so that workmen can walk on the slab. As soon as the first coat was dry, second coat was applied.
5. The next step was the placing of the heavy wire mesh sheets supported by bolsters for the ground floor slab. The mesh acts as a grid system support for fastening and wiring in place conduits, sleeves, and prestressing cables. Then the post-tensioning system of prestressing cables was placed and supported by bolsters and chairs. After the prestressing units were fastened in place, conduits and sleeves were placed and wired to mesh and prestressing units. The ground floor slab was then cast.
6. The same procedure described above was followed for the first, second, third, main roof, and solarium roof slabs.

7. When the slabs had attained a strength of at least 3600 psi as determined by test cylinders, the post-tensioning operation was begun. The high tensile strength wires were tensioned with hydraulic jacks at the edges of the slabs. After all slabs were pretressed and cables grouted, they were then ready for the lifting operation.

LIFTING OPERATION

One unusual feature of this project is the method of connecting the collar castings to the columns as shown detailed in Fig. 11. The lift collar castings

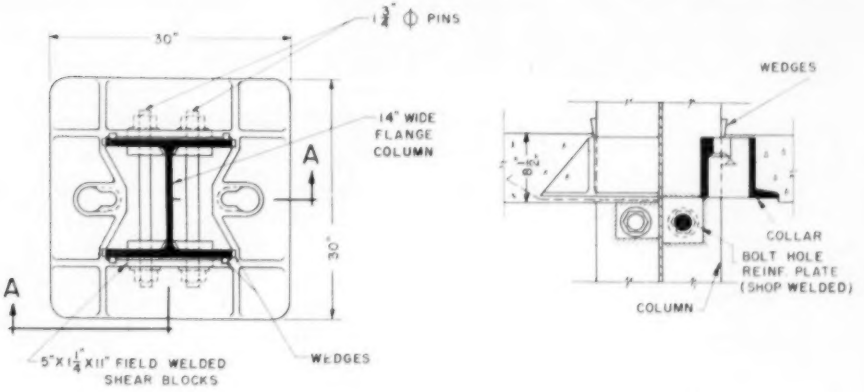


Fig. 11—Detail of column lifting collar

were initially placed over each column and the concrete slabs cast so that the collars engaged the slabs. Lifting rods were connected to the collars, and the slabs were lifted into place. When slabs are lifted to their final positions, the usual procedure has been to hold the slabs with the lift equipment until steel plate shear blocks, which normally support the slabs, are welded to each column. This time-consuming procedure ties up the lifting equipment during the welding operation. On this project the slabs were lifted into place and 1¾ in. diameter pins fastened through specially designed shear blocks to pick up the total load of the slabs. With this procedure it was not necessary to hold the slabs with lifting equipment while the welding was being done. Welding was accomplished, in general, after all slabs were lifted.

In order to provide a rigid connection to the column at the top of the casting, steel wedges as shown in Fig. 11 were driven between the castings and the columns and welded in place. The pinned shear plates at the bottom of the castings were welded to the collars and columns to insure rigidity at this point. This method of fastening collars to the columns reduced welding and lifting time considerably.

Following is the planned procedure listing the steps followed during the lifting operation:

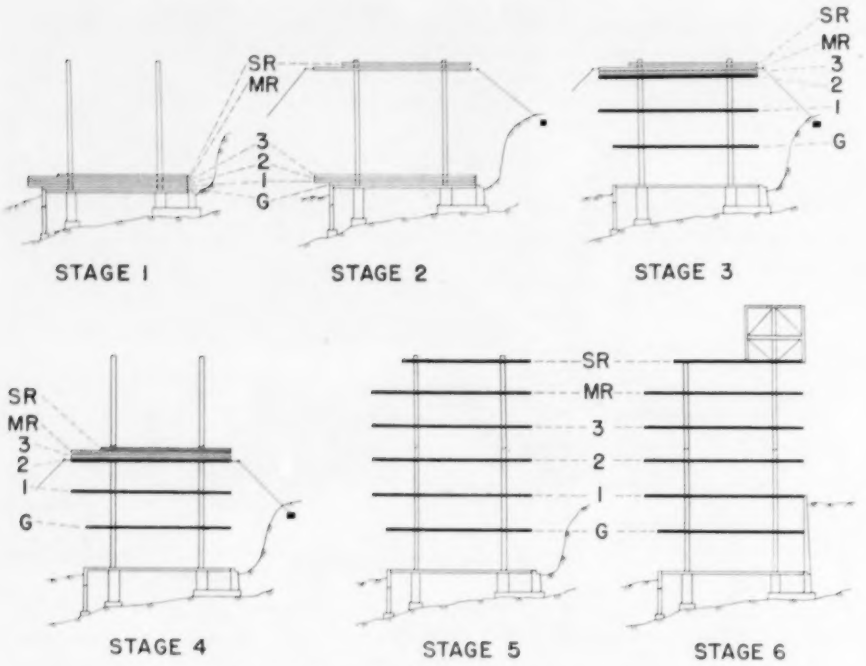


Fig. 12—Six stages of the lifting sequence

1. Stage 1 of Fig. 12 shows the construction at the start of the lifting. After the jacks were placed on top of the lower column halves and the lifting rods threaded into the casting lifting holes, the solarium roof was the first slab to be raised. The shear blocks were pinned near the top of the lower column halves at their temporary position and wedges were driven into place from below and tack welded. During the lifting of this slab and the others, a check was made and it was noted that there was a maximum of 7/8 in. horizontal displacement at one end of the slab while the slabs were en route to their position at the top of the lower columns. Further check indicated that at Stage 3 the columns were again plumb.

2. The main roof slab was then raised as was the solarium roof slab, shear blocks pinned and wedges installed from below. Fig. 13 shows solarium roof slab in temporary position at top of lower column halves and main roof slab raised a few inches off of the third floor slab. All wedges were lightly tack welded in place. During the lifting of this slab a constant check was made of horizontal displacement.

3. Four bracing cables were then fastened simultaneously to anchors which were embedded in the main roof slab at the four corners of building as indicated in Stage 2 of Fig. 12. The other ends of these cables were adequately secured to four concrete embedments in the ground. Turnbuckles on each of the four cables were simultaneously tightened sufficiently to be only slightly more than snug. During the tightening a check was made on the slab to insure that there was no horizontal displacement from its normal lifted position, with the men on each cable in almost instantaneous coordination with the instrument check. A check was made twice daily on tightness of the bracing cables to insure proper tension.

4. The third floor slab was then raised in the same manner as the solarium and main roof slabs, and shear blocks were pinned.

Fig. 13—Solarium roof slab at temporary position

5. Second floor slab was raised to correct final position, and shear blocks were pinned and welded.

6. First floor and ground floor slabs were raised to correct final positions, shear blocks pinned, wedges installed, wedges and shear blocks welded as shown in Stage 3 (Fig. 12).

7. Lifting jacks were then removed from column tops while welding progressed on ground, first, and second floor slabs.

8. As soon as shear block welds were completed for ground, first, and second floor slabs, and wedge welds were completed for ground and first floor slabs, bracing cables were transferred simultaneously from main roof slab to second floor slab under constant instrument check, and cables adequately resecured and tensioned simultaneously as in step 3 above.

9. Next, the column extensions were erected and plumbed, and the splices welded as indicated in Stage 4 of Fig. 12. Particular care was taken in welding at the splices so that shrinkage did not cause columns to be misaligned or out of plumb. Welders were placed on opposite flanges of columns, welding simultaneously. Surprisingly little difficulty was encountered. Maximum out of plumb noted on any column during welding was $\frac{1}{8}$ in., which disappeared when welds cooled. See splice detail in Fig. 14.

A continuous horizontal check in both directions on all columns was made with an instrument and any deviations from plumb were adjusted.

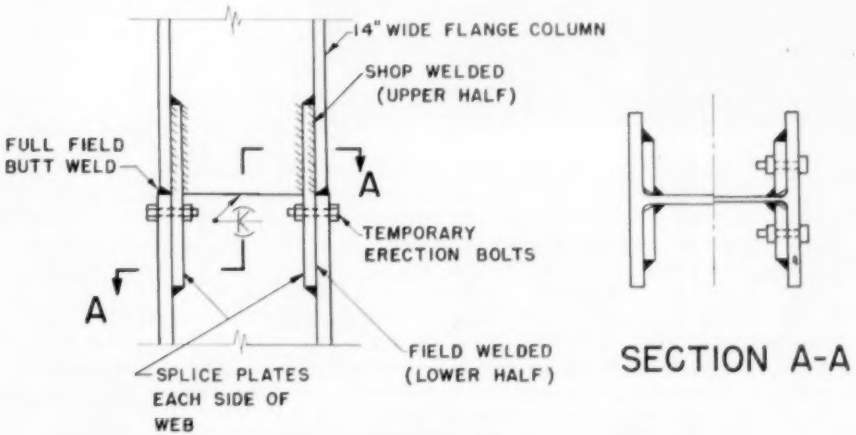


Fig. 14—Column splice detail

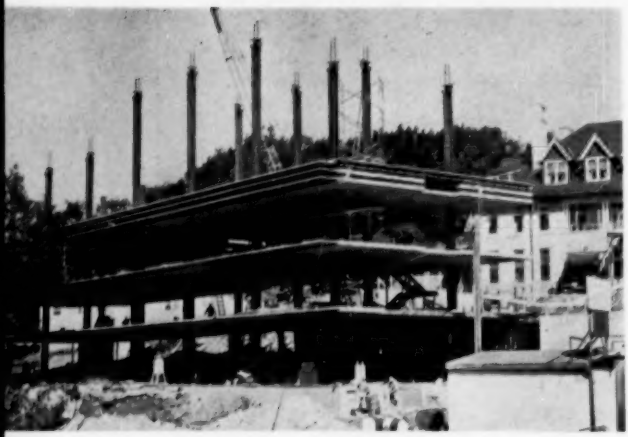


Fig. 15—Beginning of the second lifting operation, Stage 4

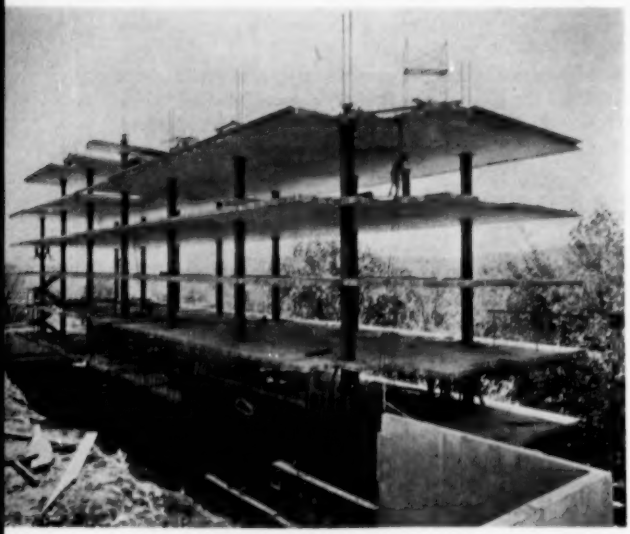


Fig. 16—All slabs in place, Stage 5

10. Lifting jacks then were placed on top of the extended columns (Fig. 15).

11. The solarium roof slab was then raised to final position, shear blocks pinned, wedges installed, wedges and shear blocks welded.

12. Main roof slab and then third floor slab were raised to final positions, pinned, wedged, and welded as shown in Stage 5, Fig. 12. This stage can be seen in Fig. 16.

13. Second floor slab wedges were installed and welded from top.

14. Lifting jacks were then removed while welding operation on upper slabs was concluded.

15. The tops of steel columns were burned off slightly below the roof slab surfaces and cap plates welded to column tops to receive insulation and roofing. Penthouse framing was then installed as indicated in Stage 6 of Fig. 12.

16. The cable bracing was maintained in place until the masonry end walls were constructed up to the second floor level.

ADVANTAGES AND DISADVANTAGES

During the design stage preliminary cost estimates indicated a saving in both the prestressing and lifting operations in comparison to conventional methods of construction utilizing mild steel reinforcing and flat plate concrete work formed in place. We are listing below both the advantages and the disadvantages of the system as designed in comparison to the conventional.

In the cost of construction there were some savings in the prestressed lift slab over the conventional formed flat plate design, as indicated in the sq ft costs below:

Prestressed lift slab		Concrete flat plate	
Prestressing complete	$0.80	Mild steel	$1.05
Mesh	0.08	10½ in. concrete slab	0.60
Extra, mild steel	0.15	Formwork	0.55
8½ in. concrete slab	0.50		
Lifting complete	0.37	Total, per sq ft	$2.20
Lifting collars	0.10		
Total, per sq ft	$2.00		

The prestressed lift slab required only an 8½ in. thick slab while a concrete flat plate design, which would accommodate the large spans and cantilevers, would require a 10½ in. thick slab, so that there was a saving in building height with the system as designed. The prestressed construction can accommodate much larger slabs accompanied by practically no deflection than can conventional construction.

Insofar as the cost of the structural design is concerned as well as the amount of field supervision required, the designing engineer should materially increase his fee for the prestressing design work in comparison to the conventional design. The calculations and drawings required for the prestressing operation are considerably more than for the conventional flat plate work.

Exposed concrete slabs require much less finishing when slabs are cast on steel-troweled finished floor slabs in the lift-slab design. Ceiling finish is equal to floor finish. The slabs on this project parted very well, there being no sticking of slab surfaces. The concrete floor and ceiling finishes were excellent, requiring a minimum of work. The prestressed slabs being in compression insured crack-free concrete.

One disadvantage of the lift-slab system, according to the contractor, was the delay in getting masons on the job. If a formed system were employed the masonry end walls and lower floor masonry partitions could have been started earlier. Concrete and reinforcement placement took about a week per slab, a total of 6 weeks for six slabs. Lifting and splicing time was 3½ weeks.

Formwork was almost entirely eliminated in the lift-slab work. Edge forms only were necessary and they were reused on each of six slabs.

Electrical conduits, sleeves, reinforcing, and concrete are more easily placed at ground level with the lift-slab operation where workmen can work more efficiently than they can on formed-in-place construction.

ENGINEERS AND CONTRACTORS

Sherwood, Mills and Smith, Stamford, Conn., were the architects; Marchant and Minges of West Hartford, Conn., were structural, mechanical, and electrical engineers for the hospital. The general contractor was A. F. Peaslee, Inc. of Hartford, Conn. Consultant on prestressing was Freyssinet, Inc., New York. Lift-slab contractor was New England Lift Slab Corp., Boston, and the prestressing units were supplied by Intercontinental Equipment, Inc.

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Title No. 53-41

From a symposium sponsored by ACI Committee 209

Shrinkage and Temperature Stresses in Masonry*

By R. E. COPELAND†

SYNOPSIS

Various methods, including crack control joints, have been proposed for crack control in concrete masonry walls due to shrinkage and temperature stresses. This paper outlines a theory based on experimental studies and experience for determining the maximum permissible distance between control joints in blank walls containing no "stress raisers." The effect of variables such as type and degree of restraint and properties of the masonry units is discussed.

The theory is offered not as a rigorous mathematical treatment of the problem but as an approximate guide for the design engineer and architect.

The subject of shrinkage and temperature stresses in masonry walls involves many separate phases, variable factors, and complexities. It does not lend itself to a neat mathematical solution. We know that masonry that is trying to contract due to internal changes, but is fully or partially prevented from doing so, will be stressed in tension and usually also in shear. We know that cracks form when these stresses exceed the strength of the masonry. Beyond this point we emerge into the field of theoretical speculation illumined only slightly by experimental data and systematic observation.

The principal approaches to minimizing shrinkage and temperature cracking in concrete masonry include: (1) improving the dimensional stability of the concrete itself and of the masonry; (2) reducing residual shrinkage by the use of properly dried units; (3) controlling crack width by means of reinforcing; and (4) incorporating wall design details which allow the masonry to contract more freely and without becoming overstressed.

This paper will discuss only one aspect of the fourth type of approach; specifically, the spacing of crack control joints in blank walls of uniform cross section and with no chases or other stress concentrating details.

CONTROL JOINT SPACING—METHOD 1

The need for information on control joint spacing has been evident to all who are familiar with current practices in using this crack remedial measure and with the results which have been obtained in buildings. Spacings usually

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†Member American Concrete Institute, Director of Engineering, National Concrete Masonry Association, Chicago, Ill.

have been selected arbitrarily with little or no technical basis. More often than not control joints are spaced too far apart with the result that cracks develop between joints. In fewer cases, the designer goes to the other extreme, using too many joints, too close together, thus unnecessarily adding to construction and maintenance costs.

In developing a theory for control joint spacing it is necessary to consider several factors and their interrelationships. It also will be necessary to make certain assumptions, some of which admittedly are not strictly valid.

Considering the masonry units, we are concerned with their shrinkage properties, coefficient of thermal contraction, ultimate tensile strain or extensibility, and their ultimate tensile strength. Regarding walls of these units, we are concerned with the variations in moisture and temperature, the location of external restraints and their stiffness relative to the stiffness of the walls and whether the restraints are subject to drying and temperature contraction prior to or simultaneously with the wall contractions. Finally, we are concerned with the distribution of stress in the wall as the masonry tries to contract against the resistance offered by the restraints. Internal restraint will be neglected. It will be more accurate but not essential to include the effect of mortar joints on the elastic and shrinkage properties of the masonry.

End restraint

Consider first a wall restrained at the ends only, as shown in Fig. 1. As the wall shrinks it will be stressed in tension, and if the stress increases enough to exceed the strength of the masonry (or if restrained shrinkage exceeds

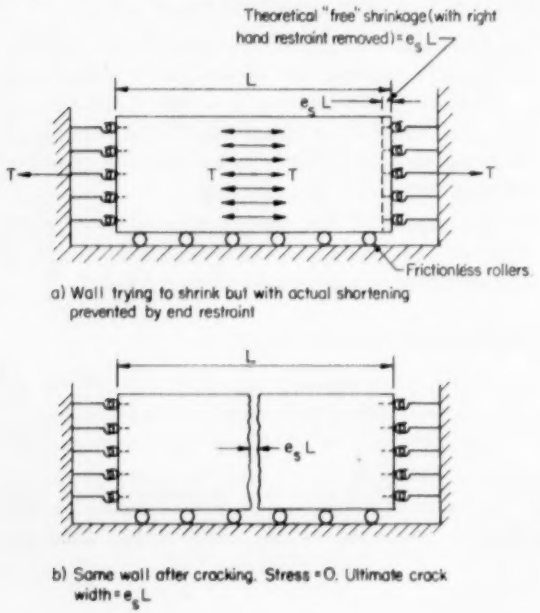

Theoretical "free" shrinkage (with right hand restraint removed) = e_s L
L
e_s L
T
T T
T
Frictionless rollers
a) Wall trying to shrink but with actual shortening prevented by end restraint
L
e_s L
b) Same wall after cracking. Stress = 0. Ultimate crack width = e_s L

Fig. 1—Effect of end restraint on stress development and cracking in walls due to wall shrinkage

the ultimate tensile strain) a crack will form and instantly the two portions of the wall become completely relaxed and can shrink at will. Crack width will continue to increase until the masonry has fully shrunk and will attain a value equal to the free unit shrinkage times the wall length. It should be noted in this case that the tensile stresses are uniformly distributed throughout the wall section and that after cracking has occurred there is no residual stress in the wall. The tendency to crack is not influenced by the geometrical proportions of the wall, only by the degree of restraint and the ratio of extensibility to free shrinkage and temperature contraction.

Edge restraint

Pure end restraint is rarely encountered in building walls, the common form being edge restraint from the foundation, floors, and roof as shown in

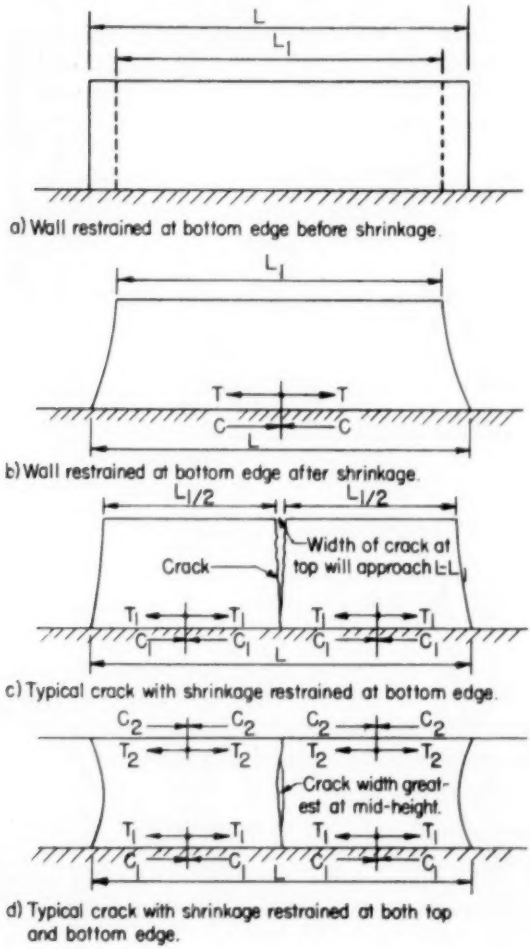
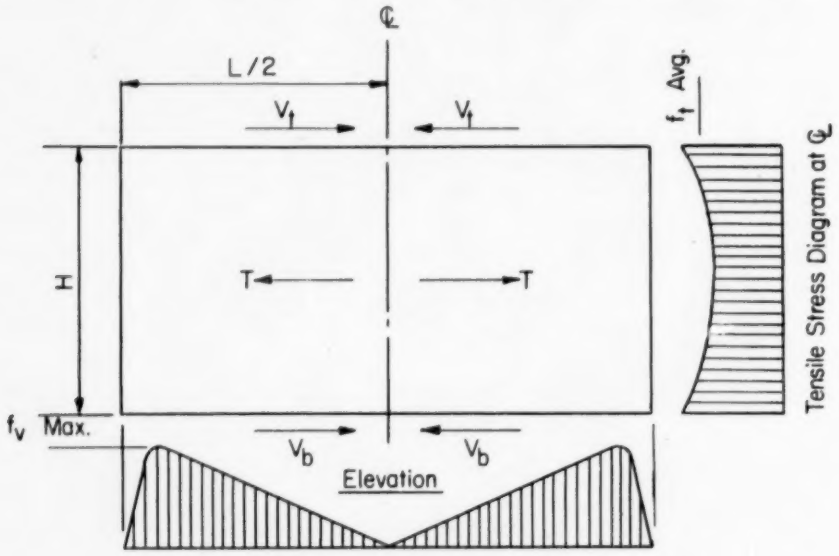


Fig. 2—Effect of edge restraint on stress development and cracking due to wall shrinkage. Full restraint at restrained edge is assumed in each case

Fig. 2 or a combination of the two types of restraint. Edge restraint differs from end restraint in action and effect in several important respects. It induces shearing as well as tensile stresses. It prevents complete relaxation of the masonry with cracking, thereby reducing crack width. It influences the number, spacing, width, and direction of cracks.

Fig. 3 is a simplified illustration of the horizontal forces and stresses acting on a wall which is free at the ends but is restrained against shrinking along the top and bottom edges. As indicated in the shear diagram, shearing stresses at the restrained edges vary from zero at the extreme ends of the wall



Cracking will occur when the length of the wall exceeds:

$$L = 2\left(\frac{f_t}{f_v}\right)\left(\frac{A_t}{A_v}\right)H$$

where:

f_t = Average tensile unit strength of wall, psi

f_v = Maximum shearing unit strength of mortar joint, psi

A_t = Wall area effective in resisting tensile stress, per foot of wall height, sq. in.

A_v = Joint area effective in resisting shear per foot of wall length, sq. in.

Fig. 3—Simplified illustration of horizontal tension and shear due to shrinkage in wall restrained at top and bottom edges

to a maximum a short distance in from the end, then decrease to zero at the centerline. The ascending and decreasing curves of the shear diagram are not linear as shown but for our purpose it is not essential to know their exact form.

With edge restraint, tensile stresses will vary from a maximum at the restrained edge to a minimum at some distance from the restraint, in this case at mid-height of the wall since equal restraint at top and bottom has been assumed. At the center line of the wall the major principal stresses (combination of tension and shear) will be horizontal or nearly so. The total horizontal tension must equal total horizontal shear.

The tensile resistance of the wall will be equal to the tensile unit strength times the cross-sectional area effective in resisting tension, assuming no transfer of stress across vertical mortar joints. The opposing force, or shear, will be equal to the shearing unit strength times its effective area.

Equating these forces gives the equation shown in Fig. 3 indicating the wall length at which tensile stress will become critical. The ratio, A_t/A_v, will be about 0.50 in hollow block construction and tensile unit strength, f_t, may be taken at 10 percent of the compressive strength of the concrete or from 150 to 250 psi for commercial quality block. There are few data on the shearing strength of mortar joints especially in walls carrying compressive loads. Some data indicate values of about 90 to 100 psi.

Inserting assumed values of 200 psi for f_t and 100 psi for f_v and solving for L in terms of H, it appears that blank walls whose length exceeds about two times their height will be subject to tensile cracking if fully or highly restrained at top and bottom edges and if unit shrinkage and temperature contraction relative to the restraints exceeds unit extensibility.

CONTROL JOINT SPACING—METHOD 2

While the above approach provides a rough rule for control joint spacing it does not properly take into account the effect of variations in the shrinkage, thermal contraction, and extensibility of the masonry. Accordingly we will consider briefly a method which includes the effect of these properties. It is based largely on unpublished experimental data developed by Roy W. Carlson and Thomas J. Reading in studies conducted for the Portland Cement Association on the distribution of shrinkage stresses in walls.*

Fig. 4 shows contour lines giving the magnitude of major principal stresses as a percent of the major principal stress at the restrained base of walls of varying ratios of length to height. From these contours it will be noted that the average stress at the center line of the wall increases markedly as the L/H ratio increases from one to four. This effect is also seen in Fig. 5, which shows how a control joint functions to decrease wall stress by decreasing the L/H ratio.

The Carlson-Reading studies of the effect of the L/H ratio pertained to walls restrained only at the base, but in Fig. 6 the data have been adapted

*Permission of the Portland Cement Association to use certain data from the Carlson-Reading experiments is gratefully acknowledged.

to walls restrained at top and bottom (upper curve) as well as at the bottom edge only (lower curve). This figure shows the relation between the L/H ratio and the average stress at the center line of the wall.

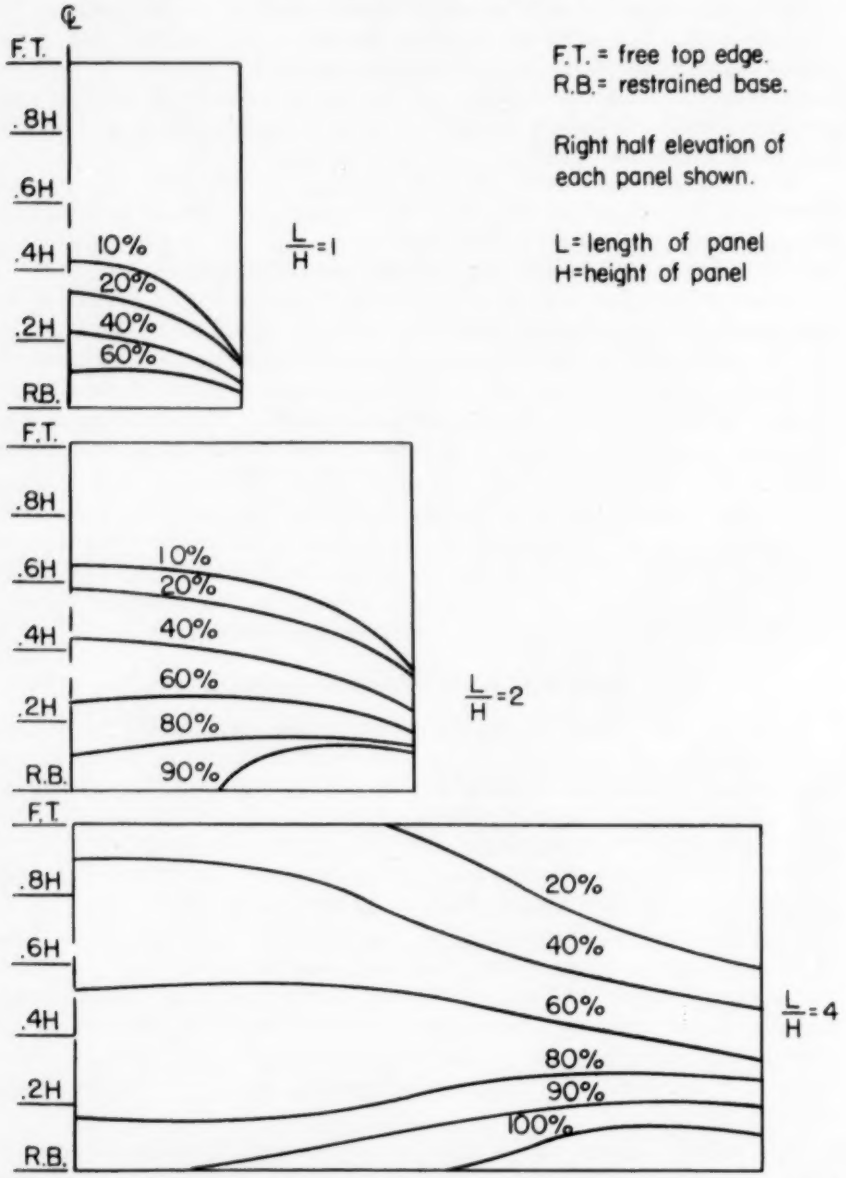


Fig. 4—Contour lines giving magnitude of major principal stresses as percent of major principal stress at restrained base, for walls of varying ratios of L/H

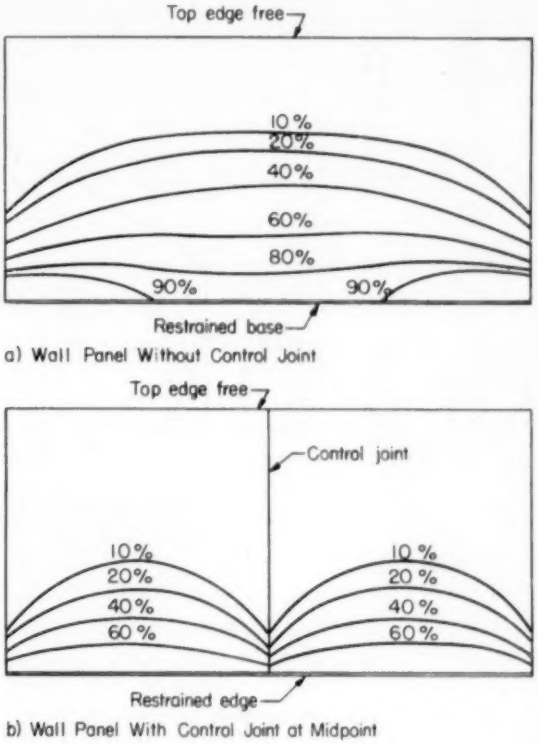


Fig. 5—Effect of control joints in reducing shrinkage stresses in walls without openings restrained at bottom edge only —contour lines indicate principal stresses as percent of principal stress at restrained base

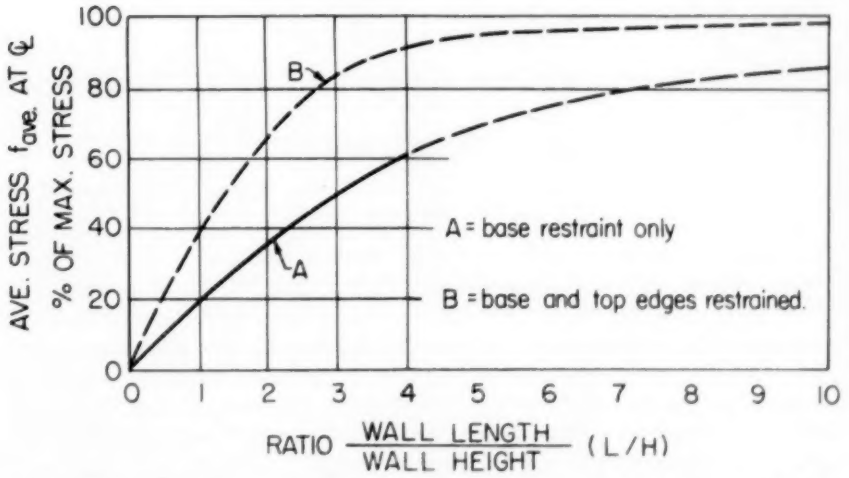
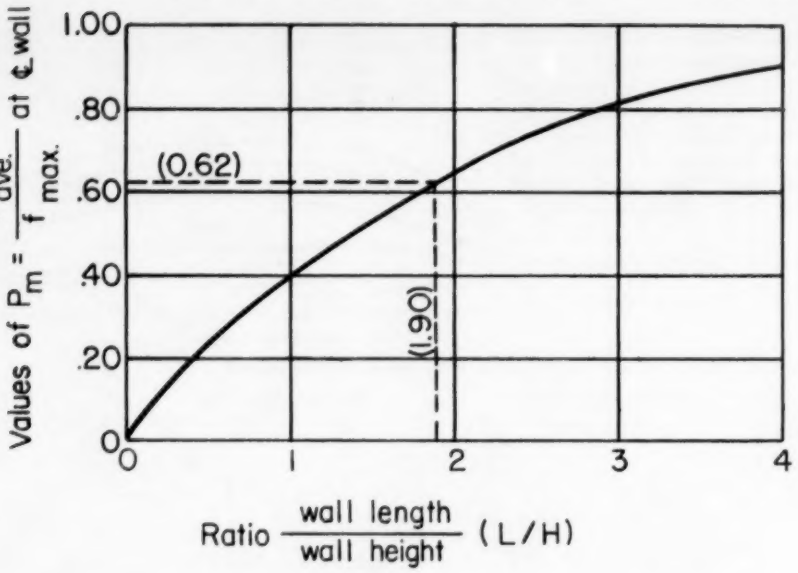


Fig. 6—Relation between average stress and maximum principal stress at restrained edge for walls of various L/H



Example, assume

$R = 0.8 \qquad e_u = 0.0002$

$e_d = 0.0002 \qquad e_f = 0.0002$

Substituting in $P_m \leqq \frac{e_u}{R(e_d+e_f)}$ gives

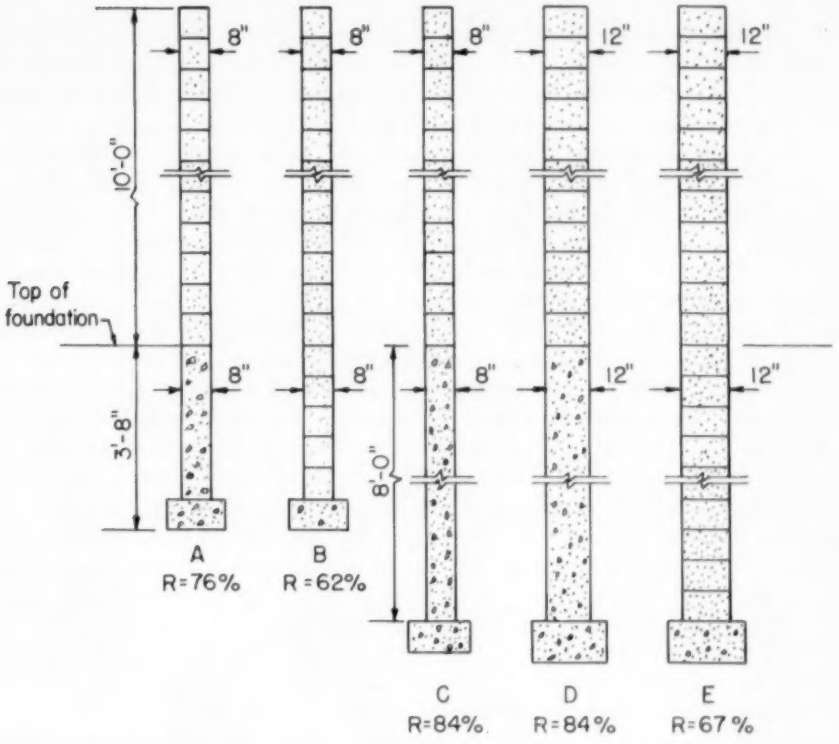
$P_m \leqq 0.62$, from which,

Wall length $\leqq$ 1.90 wall height.

Fig. 7—Example of application of control joint spacing theory, wall restrained at top and bottom edges

At this point it is expedient, pending further refinement of the theory, to make three tentative assumptions. First, that strain distribution is similar to stress distribution. Second, that cracking occurs when the average stress or strain, rather than the maximum, exceeds the ultimate values. Third, that drying and temperature stresses or strains are additive even when one precedes the other by several months. It is believed that the error introduced with these assumptions is not inconsistent with the approximate nature of the procedure in several other respects.

It can be shown that to avoid cracking the following equation must be satisfied:



Wall Length – 40 ft.
Wall Construction – Hollow block made with lightweight aggregate.
Foundation Construction – Cast-in-place concrete or hollow block made with dense aggregates.

Fig. 8—Effect of dimensions and type of foundation on the degree of restraint to wall shrinkage

$$P_m \geq \frac{e_u}{R(e_d + e_t)}$$

where P_m = ratio, average stress: maximum stress
e_u = ultimate tensile strain, in. per in.
e_d = drying shrinkage, in. per in. (between limits)
e_t = temperature contraction, in. per in. (between limits)
R = degree of restraint, decimal fraction of unity

An example is shown in Fig. 7. Here we assume a wall with no end restraint but with equal restraint at top and bottom edges. Additional assumptions are: degree of restraint, 0.80; drying shrinkage, 200 millionths in. per in.; temperature contraction, 200 millionths in. per in.; ultimate tensile strain, 200 millionths in. per in. Substituting these values in the equation indicates that P_m should not be greater than 0.62. Locating this value of P_m on the curve we find that the corresponding limiting ratio, L/H, is 1.90. That is,

the spacing between control joints for this particular wall should not exceed 1.90 times its height or the vertical distance between major horizontal restraints. Theoretically the safety factor against cracking is one, so we might arbitrarily reduce the spacing to 1.50 times the height.

The degree of restraint can be computed by approximate methods such as suggested by Saemann, Warren, and Washa.* Some idea of the effect of type of foundation on the degree of restraint produced at the base of concrete masonry walls is shown in Fig. 8. The reduction in restraint with temperature contraction or drying shrinkage of the restraining members can be approximately evaluated and included.

Values for e_u, e_d, and e_t can be taken from existing test data for various types of block and arranged to give different combinations of conditions as illustrated in Table 1. It is important to use drying shrinkage values between initial and final moisture content limits as interpolated from curves such as shown in Fig. 9, which indicate the shrinkage-moisture loss relationship for the type of masonry unit being considered. For example, assume sand and gravel block having a total shrinkage of 0.04 percent with drying from 100 to 10 percent moisture content. Also assume that the block have a moisture content of 30 percent when laid and will dry in the wall to a final moisture content of 10 percent. From curve A, Fig. 9, it is seen that the amount the block shrink in the wall is 42 percent of total shrinkage, which is equivalent to $0.42 \times 0.0004 = 0.000168$ in. per in. The design value of e_d for the masonry would be somewhat greater because of the higher shrinkage of the mortar; in Table 1 the corresponding value is 0.00022.

Finally, it should be possible to develop a family of curves of the general character of those in Fig. 10 which can be used in selecting the joint spacing required for walls of various heights and cracking tendencies.

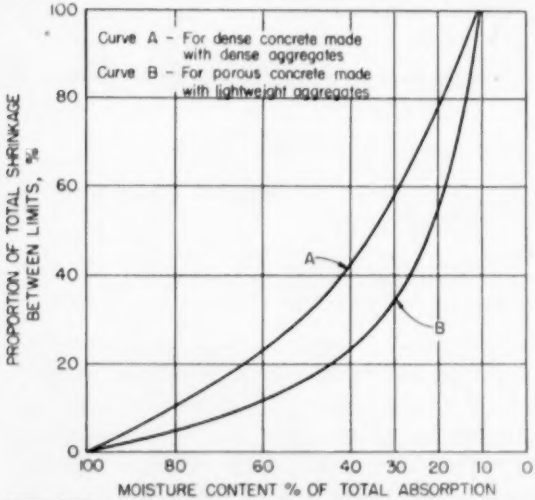


Fig. 9—Typical relation between moisture loss and percent of total shrinkage between limits of 100 and 10 percent moisture content in block cured in steam at atmospheric pressure

*Saemann, J. C., Warren C., and Washa, G. W., "Effect of Curing on the Properties Affecting Shrinkage Cracking of Concrete Block," ACI JOURNAL, May 1955, *Proc.* V. 51, pp. 833-852.

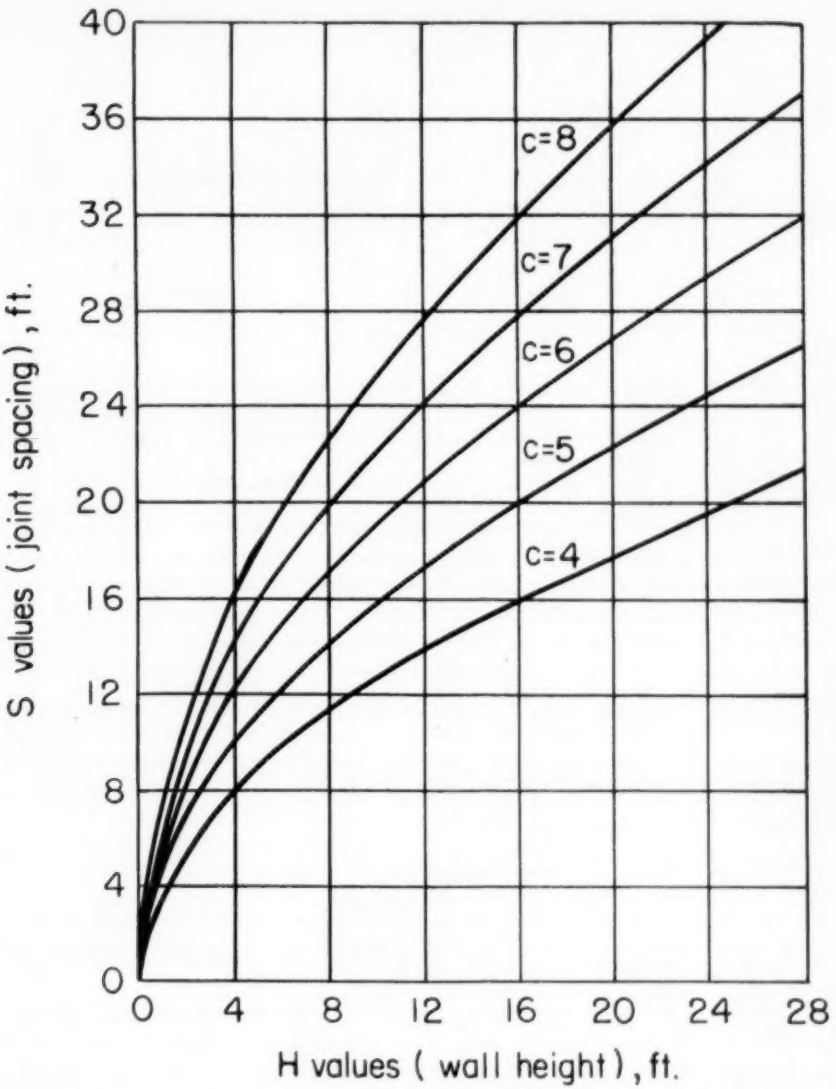


Fig. 10—Control joint spacing for blank walls with no "stress raisers," from formula, $S = C\sqrt{H}$

In the general equation for joint spacing, $S = C\ H^x$, the coefficient C will be functional to $e_u/[R(e_d + e_t)]$ at some selected value of H and the exponent x selected to allow adequately for scale or mass effect. Actually, the coefficient C would be derived to give joint spacings at the selected wall height which agree with those obtained from Fig. 6.

TABLE 1—RATIOS OF EXTENSIBILITY TO SHRINKAGE*

Block data			Values for walls					
				Shrinkage, percent†				
				Drying to				
Type of aggregate	Shrinkage class	Initial moisture content, percent	Extensibility, e_u, percent	10 percent moisture content, e_d	20 percent moisture content, e_d'	40 F drop, e_t	$\frac{e_u}{e_d' + e_t}$	$\frac{e_u}{e_d}$
Dense	Normal	60	0.020	0.038	0.027	0.020	0.42	0.52
		40		0.028	0.018		0.53	0.71
		30		0.022	0.011		0.65	0.91
	Low	—	0.018	—	—	0.020	—	—
		—		—	—		—	—
		30		0.012	0.007		0.67	1.50
Lightweight	Normal	—	0.025	—	—	0.015	—	—
		—		—	—		—	—
		30		0.063	0.022		0.67	0.40
	Low	—	0.022	—	—	0.015	—	—
		—		—	—		—	—
		30		0.033	0.012		0.81	0.67

*Values shown are for illustrative purposes.
†Shrinkage relative to restraints.

To be useful to design engineers and architects the theory needs to be further developed and put in simple and concise form. Also, it should be supplemented by information regarding the proper location of control joints in walls containing discontinuities which act as local stress raisers.

Among the desirable refinements is the development of strain contours for walls with edge restraint. Possibly this can be accomplished by proper modification of the Carlson-Reading stress contours. The assumption that strain and stress distribution are similar obviously is not true and as applied in the procedure outlined tends to give conservative joint spacings even though the selected value of e_u, ultimate tensile strain, includes plastic as well as elastic strain.

SUMMARY

To be effective in preventing cracks (not merely minimizing crack width), control joints must be properly spaced and located. Arbitrary spacings too often are either ineffective or too costly.

The problem of control joint spacing in blank walls is susceptible to approximate theoretical solution provided information is available, or reasonable assumptions can be made, regarding the variable factors involved. In this paper a theoretical method is briefly illustrated which is believed to have considerable promise, especially with further refinement. While not offered as an "exact method" or a rigorous mathematical treatment of the subject, it is based on rational relationships of the principal factors and what is perhaps equally important appears to agree rather well with some observations in the field as to crack spacing in walls with and without control joints.

Discussion of this paper should reach ACI headquarters in triplicate by June 1, 1957, for publication in the Part 2, December 1957 JOURNAL.

From a symposium sponsored by ACI Committee 209

Title No. 53-42

Some Factors Influencing Shrinkage of Concrete Pavements*

By F. N. HVEEM and BAILEY TREMPER†

SYNOPSIS

Portland cement concrete pavements tend to become rougher with time. To a degree the development of roughness is initiated by curling of the ends of the slabs. In California, at least, slabs curl upward more than downward because shrinkage due to drying is more pronounced in the upper part of the slab. When curled upward, the slabs are not supported uniformly by the subgrade and the ends deflect more under load. Under heavy traffic, cracks and faulting develop and the pavement becomes progressively rougher. The integrity and smoothness of the pavement can be prolonged if the characteristic shrinkage of concrete can be reduced.

Both portland cement and aggregates affect the amount of shrinkage. Some of the factors contributing to the influence of portland cement are known, although not adequately restricted in standard specifications. The contribution of clay in aggregates to shrinkage has not received the attention it warrants and national specifications do not guard adequately against excessive amounts of clay.

The California Division of Highways has developed simple, short field tests that evaluate both the quantity and activity of clay contained in aggregates. These are known as the "sand equivalent" and "sedimentation" tests. Data are presented to show that a high degree of correlation exists between the results of these tests and the drying shrinkage of mortar and concrete. With suitable specification limits, these tests are effective in securing important reductions in drying shrinkage.

It is not a particularly difficult matter for an engineer today to construct a pavement or any structure from portland cement concrete. When the work is completed and ready for use, the pavement or structure usually conforms to the plans and specifications. In other words, it is exactly where it should be within close tolerances. However, experience has shown that while a pavement may be constructed that is true to line and grade for all practical purposes, there is no assurance that it will stay that way. While change in contour or surface roughness or riding quality is not peculiar to any one type of pavement, one of the factors which may cause a smooth,

*Presented at the ACI 52nd annual convention, Philadelphia, Pa., Feb. 22, 1956. Title No. 53-42 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 8, Feb. 1957, *Proceedings* V. 53. Separate prints are available at 50 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than June 1, 1957. Address P. O. Box 4754, Redford Station, Detroit 19, Mich.

†Members American Concrete Institute, Materials and Research Engineer and Supervising Materials and Research Engineer, respectively, California Division of Highways, Sacramento, Calif.

relatively perfect concrete pavement to become rough and wavy within a few years is the shrinkage of the concrete.

EXAMPLES OF PAVEMENT CURLING

Fig. 1 represents a profilogram illustrating typical changes and movements in concrete pavement slabs that are of daily occurrence.

Fig. 2 is a profilogram showing another pavement where early morning and afternoon recordings were made at intervals during the life of the project. The profilograms represent the pavement 2 months, 1 year, and 2 years after construction.

Fig. 3 is a drawing of an individual pavement slab showing the differences in shape between the early morning and the afternoon on a warm day. The magnitude of slab deflection under a heavy truck load is also indicated. Note that the deflection at the slab end is about four times as large in the early morning when it has the greatest upward curl, and that there is no greater deflection at the end than elsewhere when the slab is "flattened down" to make contact with the subgrade.

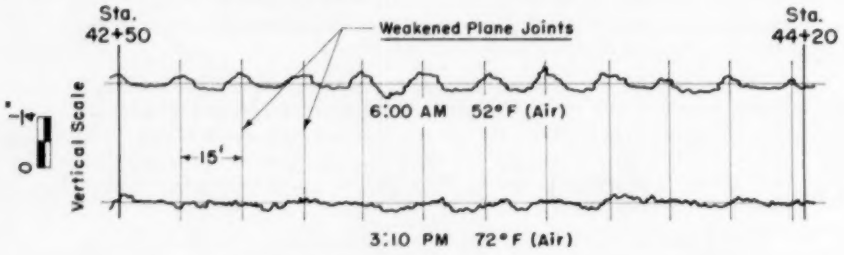


Fig. 1—Effect of temperature on profile of pavement

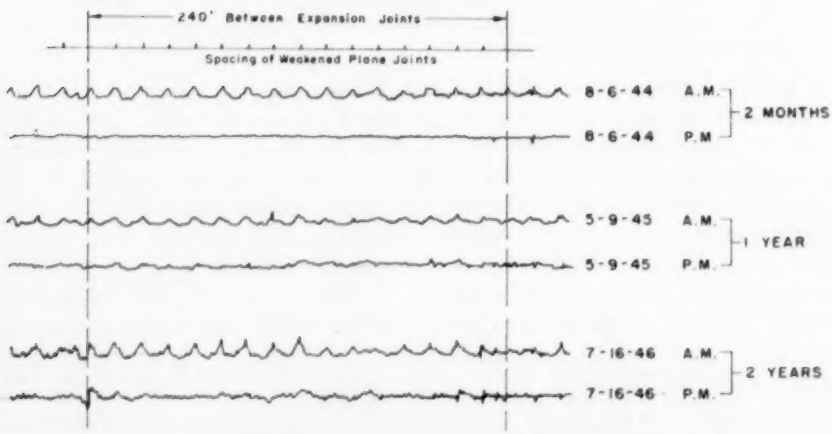


Fig. 2—Effect of time on profile of pavement

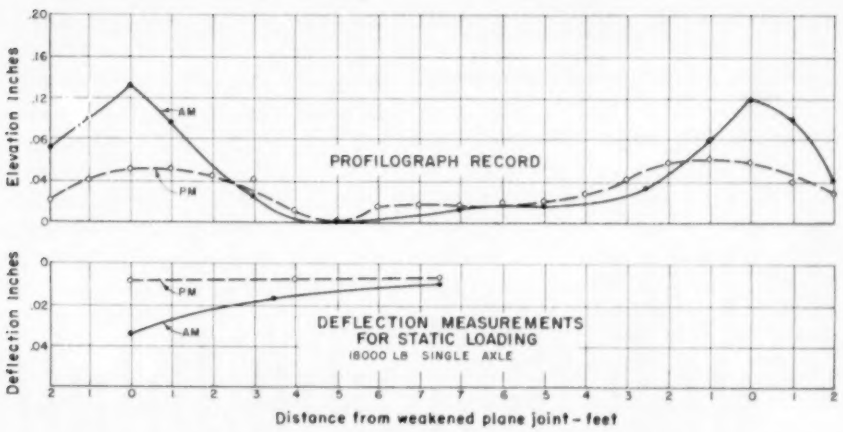


Fig. 3—Effect of curling on deflection under load

CAUSES OF CURLING

Failures near joints in portland cement concrete pavement may be attributed to one or more causes; either the subgrade is yielding under the passing loads or the slab is being deflected when not supported by the subgrade. While the character of the supporting soils is undoubtedly a factor, it seems evident that much of the roughness in concrete pavements would not develop even over a comparatively poor subgrade if the concrete pavement remained in contact with the subgrade at all times and at all points beneath the slab. This particular discussion, therefore, deals primarily with the properties and characteristics of portland cement concrete which lead to slab warping or curling.

A curled slab is an unstable mechanism that, in effect, creates a pumping machine where the concrete slab is a movable diaphragm under each passing heavy wheel load. To determine the reasons for this change in shape, an investigation was carried out on concrete pavements in California and some of the results were presented in a paper entitled "Slab Warping Affects Paving Joint Performance."[1] Fig. 8 from that paper is reproduced here as Fig. 4, illustrating that the expansion of dry concrete due to absorption of moisture is in many cases greater than the thermal expansion resulting from a 90-deg rise in temperature. The magnitude of shrinkage could be equally great.

It is obvious from an examination of the contour of the curled slabs, that either the underside of the pavement has become longer or the upper surface has become shorter. While it is probable that both forms of volume change are present, nevertheless, the underside of the slab was wet when laid and being in contact with the subgrade probably never dries out or loses an appreciable amount of moisture. The upper surface, on the other hand, does dry out and will shrink according to the characteristics of the particular concrete. When this surface shrinkage is combined with contraction due

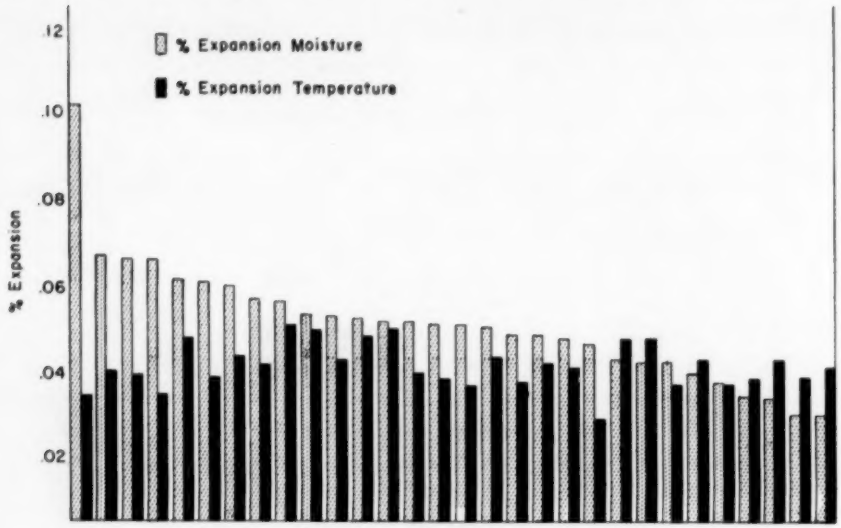


Fig. 4—Comparison of moisture and thermal expansion of concrete

to low temperatures of the upper surface, then the pavement slab will manifest its utmost curl.

The following discussion deals with some of the factors that can be responsible for abnormal shrinkage of portland cement concrete. These relationships may explain some of the differences in pavement performance which have been observed, although no data are available concerning the properties of the particular cements and aggregates used in some of the older projects.

CAUSES OF DRYING SHRINKAGE

R. W. Carlson[2] has shown that drying shrinkage is affected by many factors. Among these are the characteristics of the two major constituents, cement and aggregates.

Portland cements have been found to produce variations in shrinkage of as much as 50 percent. William Lerch[3] has shown that these variations can be reduced substantially by controlling the gypsum content of the cement to an optimum value. To date, however, a workable method of securing optimum gypsum content by means of purchase specifications has not been developed. This subject is still under study in ASTM Committee C-1 and it is hoped that a solution to this important problem will be obtained.

The influence on drying shrinkage of clay in aggregates has not received the attention it warrants. The term "clay" is here used in the layman's sense. Nationally recognized specifications do not provide adequate restrictions against clay. The specified determination of clay lumps does not include clay as coatings or clay in finely divided form. The usual test for

material finer than the No. 200 sieve does not distinguish between clay and inert fine particles.

The California Division of Highways has developed rapid field tests for the quantity and activity of clay in aggregates that correlate well with the development of drying shrinkage in mortar and concrete. These tests are known as the "sand equivalent" test which is applied to fine aggregate and the "sedimentation" test which is used on coarse aggregate. Although differing in details of manipulation because of particle size, both methods are based on the same principle. Clay, if present in the sample either as a coating or in the form of discrete particles, is brought into suspension in a dilute solution of calcium chloride which causes a moderate controlled flocculation of colloidal particles. The suspension is allowed to settle for a definite period of time and its height is measured. The particular solution used produces a greater settled volume of the more active clays such as montmorillonite than it does with kaolin for example.

SAND EQUIVALENT TEST

Fig. 5 shows the relationship between drying shrinkage and the sand equivalent of a series produced by increments of washing of a dirty pit run sand. Note that shrinkage of concrete follows trend shown for mortar.

Fig. 6 shows the relationship between drying shrinkage of mortar and the

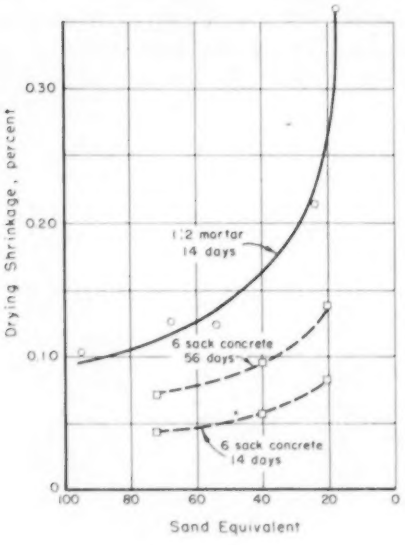


Fig. 5—Influence of sand equivalent on drying shrinkage

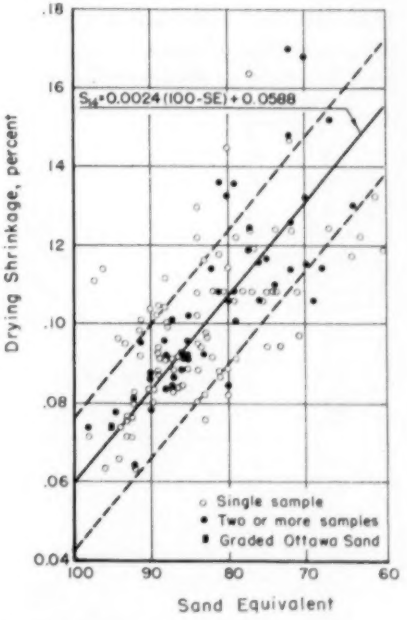


Fig. 6—Influence of sand equivalent on drying shrinkage

sand equivalent of 248 samples of commercially produced concrete sands from 142 sources of supply in California. Test specimens were 1 x 1 x 10-in. gage length bars of 1:2.75 mortar cured moist for 7 days at standard temperature and then subjected to drying in an atmosphere maintained at 100 F and 70 percent relative humidity.

The equation relating sand equivalent and drying shrinkage is:

$$S_{14} = 0.0024\,(100 - SE) + 0.0588 \pm 0.017 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where S_{14} is the percentage of drying shrinkage at 14 days, SE is the sand equivalent, and the last term is the standard error of estimate (shown by broken lines in Fig. 6).

The coefficient of correlation of this equation is 0.66 which, for the number of tests involved, indicates a probability of better than 99 to 1 that the observed relationship is not due to chance.

A substantial number of the sands produce shrinkage that departs from the general equation by more than the standard error of estimate. Many of these sands come from a few limited areas in the state. This fact indicates that performance may be related to mineralogical composition. Carlson[2] found this to be the case and concluded that the normal shrinkage of the cement paste is resisted by the aggregate, the more compressible particles offering less restraint. He also found that apparent compressibility of the aggregate is a function of porosity, and that drying shrinkage is related to the absorption of the aggregate as determined by the standard test.

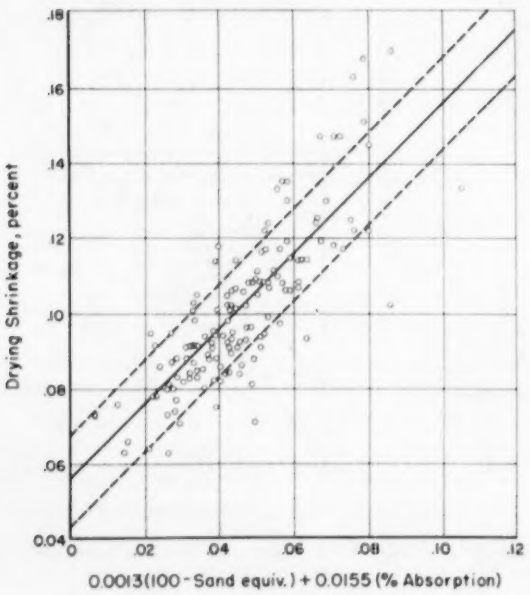


Fig. 7—Influence of absorption and sand equivalent combined on drying shrinkage

For the sands considered in the present study, the relationship between drying shrinkage and absorption is:

$$S_{14} = 0.0198A + 0.0723 \pm 0.0516 \quad (2)$$

where A is the percent absorption of the sand.

A multiple relationship between sand equivalent and absorption combined and drying shrinkage has been computed and is expressed by the following equation:

$$S_{14} = 0.0013\,(100 - SE) + 0.0155A + 0.0561 \pm 0.0121 \quad (3)$$

The coefficient of correlation is 0.83, a very significant value indicating that the contribution of the sand to drying shrinkage is dependent to a high degree on sand equivalent and absorption. Fig. 7 is a plot of the multiple relationship. Some of the points that fall well below the plotted curve represent sands of high absorption due to vesicular lava. Evidently the rigidity of such rock is not seriously affected by its pores.

SEDIMENTATION VALUE

Tests made to date to determine the effect of the sedimentation value of coarse aggregate on drying shrinkage are rather limited. Results are shown in Fig. 8. Drying shrinkage was measured on 3 x 3 x 10-in. gage length bars of 6-sack concrete, 3- to 4-in. slump, using ¾-in. maximum size of aggregate. The same cement and sand were used throughout and the coarse aggregate comprised 50 percent by absolute volume of the total aggregate.

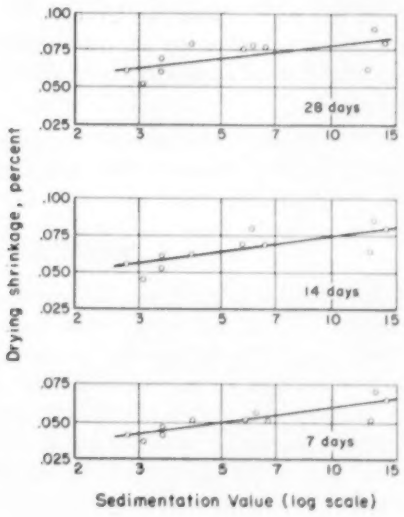


Fig. 8—Influence of the sedimentation value of coarse aggregate on drying shrinkage

The equations of the curves plotted in Fig. 8 are:

$$S_7 = 0.0333 \log SV + 0.0262 \pm 0.0059 \quad (4)$$

$$S_{14} = 0.0359 \log SV + 0.0391 \pm 0.0070 \quad (5)$$

$$S_{28} = 0.0298 \log SV + 0.0489 \pm 0.0073 \quad (6)$$

where S_7, S_{14}, and S_{28} are the percentages of drying shrinkage at 7, 14, and 28 days, respectively, and SV is the sedimentation value.

The coefficients of correlation for Eq. (4), (5), and (6) are 0.84, 0.78, and 0.70, which indicate a high level of significance.

The water-cement ratio required for equal slump increased with increasing sedimentation value and, therefore, drying shrinkage is shown to be related to water-cement ratio. Coefficients of correlation for the latter relationship, however, are relatively low, from 0.50 to 0.59. It is indicated, therefore, that the sedimentation value measures a property of the coarse aggregate that is of greater influence than the water demand alone with respect to the development of drying shrinkage.

EFFECT ON WORKABILITY

The more thorough removal of clay tends to produce lower workability in concrete. The demand for "fat" mixes with a low cement content by the building trades in California has created a problem in specifying a value of sand equivalent as high as considered desirable for highway work. The slightly higher cost of cleaner sand is not considered to be of moment, but the difficulty of securing a special sand for highway use in urban areas served by established commercial plants is a major problem.

Loss in workability, if important, can be restored by air entrainment, which has been shown[4] to have little effect on drying shrinkage. It is hoped that the dissemination of information on the adverse effects of clay will result in greater appreciation of the benefits of more thorough washing of the aggregates.

Although beyond the scope of this paper, it is worthy of note that the removal of clay as measured by the sand equivalent and sedimentation tests also results in important gains in compressive and flexural strength of concrete.

PRACTICAL SIGNIFICANCE

The practical significance of the data presented is that the more thorough removal of clay by better washing of aggregates will result in important reductions in the drying shrinkage of the resulting concrete. Reductions of up to one-third to one-half may be obtained when "borderline" aggregates are cleaned to a degree that is entirely feasible economically. A higher level of improvement is possible by also placing a limitation on the maximum percentage of absorption. But, since absorption is more or less an inherent property of the aggregate from a given source and cannot easily be reduced

by processing, the inclusion of a limitation on this property in specifications may be difficult to justify economically.

CONCLUSIONS

Drying shrinkage is an important property of concrete for all purposes. In pavements, low shrinkage is believed to be a major contributing factor to the maintenance of structural integrity and good riding qualities.

Important reductions in drying shrinkage are feasible by more thorough removal of clay in the processing of aggregates. Rapid means of estimating clay content are afforded by the sand equivalent and sedimentation tests. These are the only quick tests presently available that can be used to determine whether aggregates are really clean.

More thorough washing of aggregates may tend to result in less workable concrete, but this can be offset by air entrainment which has little effect on drying shrinkage.

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From a symposium sponsored by ACI Committee 209

Title No. 53-43

Study of Shrinkage in Concrete Frames*

By MORGAN B. KLOCK† and ROBERT R. SHERIDAN‡

SYNOPSIS

During the planning for a building at the Kodak Park Works in 1939, the question of whether to build the frame in two sections with a contraction joint was thoroughly explored. The total length was to be 275 ft; without a joint it would be one of the longest continuous frames in Kodak's Rochester plants. It was decided to construct the building as one unit and to undertake a study of shrinkage in the frame.

Carlson electric strain meters were buried in the concrete in several locations in four slabs, horizontally and vertically. These have provided most of the data which now make up a 16-year record. Drying shrinkage in the order of 500 millionths horizontally and 1000 millionths vertically has been indicated.

A limited correlation has been obtained by some direct measurements in the building, which indicate that over-all shrinkage in the frame has been roughly the same as that indicated by the meters. Some observations are made as to the relative behavior of masonry walls.

HISTORY AND DESCRIPTION OF INSTALLATION

In designing a reinforced concrete building of considerable length, the initial cost of building contraction joints into the frame must be weighed against the possible later cost of maintenance and repair of damage due to shrinkage. In 1938 we were encouraged to construct a building without joints and to study the behavior of the concrete frame through the use of Carlson electric strain meters. Our plan was to gather as much information as possible on shrinkage indicated by the meters and to correlate the data with observations of the building structure.

The frame was cast during the summer of 1939. It was of flat slab construction, 275 ft x 131 ft and seven stories high. The bays were 21 ft 4 in. x 22 ft. The slabs were 9 in. thick, designed for a 300-lb live load, according to the ACI Code.

Six standard strain meters whose gage length was 10 in. were placed in the slabs, each at the center of a bay, parallel with the long axis of the building, and half-way from the compressive face of the slab to the plane of the reinforcing steel. They were set on the second, fourth, sixth, and seventh floors,

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†Member American Concrete Institute, Structural Engineer Consultant, Eastman Kodak Co., Rochester, N. Y.
‡Structural Engineer, Eastman Kodak Co., Rochester, N. Y.

in interior and exterior bays. In addition, two meters 6½ in. long were placed vertically, one each on the second and seventh floors, about 6 in. from one of the horizontal meters. Changes in length and in temperature were determined by a Wheatstone bridge reading set. For a description of the meters and the method of taking readings, the reader is referred to *Measurement of Structural Action in Dams*, by J. M. Raphael and R. W. Carlson, James J. Gillick & Co., Berkeley, Calif., 1954, pp. 5, 51, 57.

The lead wires were run down through the formwork so that readings could be taken immediately. These wires were used until the building was completed. In the meantime, conduits were run from the meters to a switch box at a convenient location. Since the time of occupancy of the building, all readings have been taken here, with corrections made to eliminate the effect of the long lead wires.

PROCEDURE

The meters were placed in the slabs shortly after the area had been screeded. One-half hour after setting them and before the initial set of the concrete had occurred, the first reading was taken, which was considered in each case to be the neutral reading for the series.

Readings were taken daily for the first few weeks, then gradually less often and less regularly as the age increased. There were, of course, many irregularities at the beginning, but conditions of temperature and moisture gradually became more uniform as the building was enclosed and then heated. From the time of occupancy, say at about age 18 months, much of the building has been air-conditioned, so that changes in temperature have been relatively small.

The information obtained directly is temperature and indicated change in length. A correction for the temperature of the meter then produces actual change in length of the concrete from all causes. A further correction for temperature of the concrete yields the value which we have considered to be drying shrinkage. Another factor based on flexural stresses is undoubtedly present, but the amount is thought to be small.

ANALYSIS OF DATA

These data indicate a general trend in shrinkage which we feel substantiates several deductions as to the behavior of this concrete.

Fig. 1 shows the average drying shrinkage as measured by the six horizontal and two vertical meters. At the end of 16 years, the shrinkage vertically is 985 millionths, and horizontally it is less than half, or 460 millionths. The 985 millionths is almost 0.1 percent. Horizontally the amount is equivalent to 0.55 in. per 100 ft.

The difference between vertical and horizontal shrinkage is attributable to the restraint of the mat of reinforcing steel in the slabs, which is a relatively small factor, and the resistance of the building columns to bending. This

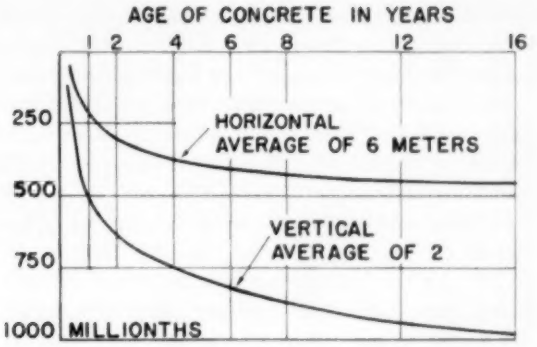

Fig. 1—Drying shrinkage in concrete building

would result in many small cracks across the slab, or possibly a few noticeable cracks at points of weakness, such as construction joints. The average restraint indicated by this difference is 0.13 in. per bay.

Fig. 1 also shows that one-half the shrinkage has taken place in the first year, three-quarters of it in less than 4 years, and 90 percent in about 6 years horizontally and 8½ years vertically. This last difference and the flatter curve indicate that restraints have stabilized the dimensions horizontally but not vertically. Right now the evidence is inconclusive whether the shrinkage has stopped: the 1955 readings show a slight expansion from the preceding set, but the amount is within observed variations from mean curves (Fig. 1).

Fig. 2 shows the record of experience of one horizontal meter during the

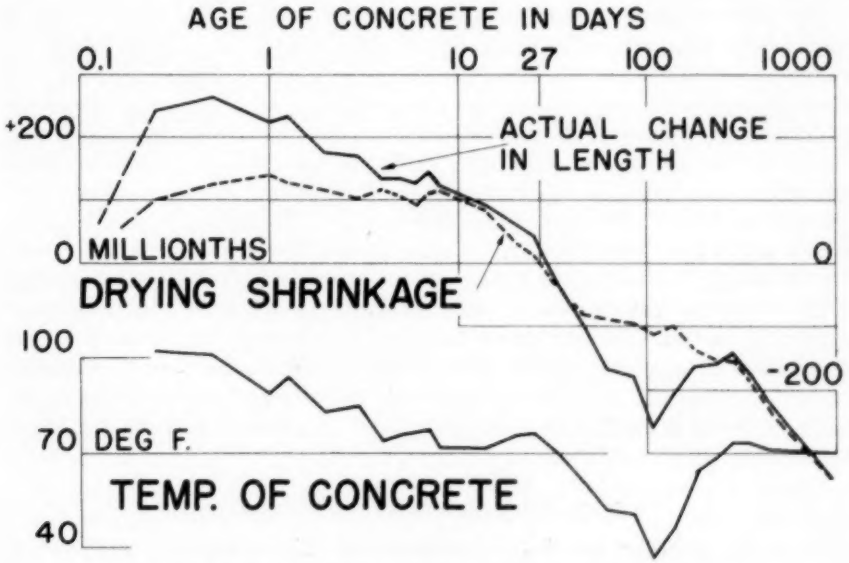

Fig. 2—Typical record for horizontal meter during first 3 years

first 3 years. The specific details of the pattern at early ages varied considerably among the meters, but this one is about average. The three curves are actual length change, temperature, and drying shrinkage.

It is plain that temperature is a large factor in volume change, and at early ages the predominant factor. During the first 24 hr, the concrete expanded about 250 millionths, mostly due to the rise in temperature. This slab was cast in July, and evidently the air was warm enough to prevent rapid dissipation of the heat of hydration. The last slabs were cast near the end of September, and the initial expansion was much less. The curve of length change follows the temperature curve for the first few months, indicating this to be a larger factor than shrinkage.

The shrinkage curve also shows an early expansion, somewhat slower to develop, which held fairly constant for a few days, then reduced gradually, but did not return to the original length until the 27th day. From the time the building was closed in and some heat applied, drying shrinkage has been practically the only variable and the two curves are substantially the same.

To provide a yardstick for comparing temperature and drying as causes of changes in length, this observation is useful: the 460 millionths of drying shrinkage which has taken place in 16 years is the same amount which would occur if the temperature were to drop 100 F.

LOCAL AND SHORT-TIME IRREGULARITIES

The irregularities in this record should be pointed out, whether they have been satisfactorily explained or not. Perhaps some readers may find a significance which has escaped us in these apparent inconsistencies.

The amount and the period of early expansion (total change from all causes) varied from 30 to 260 millionths and from 2 to 35 days, except that one meter indicated an immediate shrinkage from the original reading. It is obvious that variations in rainfall and temperature after each concrete placement affect the slabs materially, but these factors do not resolve the differences.

The curves of drying shrinkage, with one exception, indicate a similar expansion varying from 15 to 200 days. Rainfall on the fresh slabs would cause some delay in the start of drying shrinkage. After this, the only moisture added would have been due to an increase in relative humidity during plastering of walls. Otherwise, we have not accounted for this delay in shrinkage.

There are reversals in the drying shrinkage curves, which definitely were caused by rainfall while the frame was open to the weather. There is a good correlation of simultaneous reversals in several slabs, occurring 1 or 2 days after moderate to heavy rains. However, there have been reversals in the order of 20 to 50 millionths throughout the course of the study for which we have found no explanation.

CYCLES OF VARIATION NOT ESTABLISHED

Since the building has been maintained at fairly constant temperature and relative humidity for many years, it might be expected that the latter

part of the record would be uniform. This has not been the case. Individual readings of drying shrinkage have differed from the mean curve for that meter by as much as 80 millionths for no obvious reason. The question arose whether there could be a daily, a weekly, or other periodic cycle of movement in the building, due possibly to external causes. To test this idea, several series of readings were taken in the morning and afternoon for several consecutive days; they were taken in January, April, July, and November; and one group of readings was taken hourly for 24 hr. They have shown no cycling and no regular pattern.

CORRELATION OF METER READINGS WITH BEHAVIOR OF THE BUILDING

There have been only a few opportunities for direct measurement of known distances in the building. A program for such measurements was planned at the beginning and steel plugs were set in the slabs. However, the occupancy of the building has made many of these base lines inaccessible. The checks which have been possible are summarized in Table 1. The first comparison is fair, the next two are good, and the fourth is poor. But there is a fair indication that movement of the building frame has been of the magnitude indicated by the meters.

Recent inspections of the building have shown no bad cracks in the floors. The construction joints have opened up, but not by an abnormal amount. We feel it is right to say there has been no evidence of distress in the building frame.

TABLE 1—CORRELATION BETWEEN METER READINGS AND MEASUREMENTS OF KNOWN LENGTH*

Distance taped, ft	Interval between readings, days	Shrinkage†	
		Measured by tape	Indicated by meters
243	5392	220	155
128	5833	340	388
246	20	40	40
246	57	70	8

*All data reduced to 70 F.
†In millionths.

OBSERVATIONS OF SHRINKAGE IN OTHER KODAK BUILDINGS

Building 50 at Kodak Park, built in 1917, is 560 ft long, of beam and girder construction, with one contraction joint near the middle. The joint was made by forming brackets at the end of the first section, covering these with roofing paper, then supporting the beams of the second section on the brackets. The beams were not free to slide on the brackets, so that as the sections contracted many of the brackets were cracked. The trouble began about the time the building was completed, and continued for more than 20 years. The joint has opened about 1 in.

Building 15 at the Camera Works plant, built in 1939, is a flat slab building, 300 ft long with no joints. It has a low parapet wall of monolithic concrete, well anchored in the roof slab, with a terra cotta coping. Shortening of the frame without comparable shrinkage of the coping caused the latter to buckle

and slide outward at one end, after about 7 or 8 years. The exterior walls are of hollow tile, with a veneer over the columns. At the corners, this veneer is carried around on a 5-ft radius. When the building was several years old, a large crack developed at both ends near the corners evidently caused by differential shrinkage.

Building 9 at Kodak Park, was built in 1949, a flat slab building, 340 ft long, without a contraction joint. There has been no trouble with the concrete or masonry on account of shrinkage. Because of previous experiences with the cracking of walls in Building 15, walls of Building 9 were built somewhat differently. They are made up of 8 in. of hollow tile laid between columns, with a 4-in. brick facing which extends across the face of the columns as a veneer. To allow for some movement between concrete and masonry, a cushion of glass fiber board was placed between the tile and the sides of the columns. This does nothing to prevent or provide for cracking at the corners, but no such cracking has yet occurred.

CONCLUSIONS

1. Even under stable conditions of temperature and moisture, shrinkage in a concrete building frame continues over a long period, being measurable at least through 16 years, and in this instance approaching 0.1 percent as indicated by the vertical meters.

2. In this building the shrinkage horizontally has been only one-half the amount of the free shrinkage vertically. The difference is due probably to a combination of the restraint of the reinforcing steel in the slabs (a small factor) and the resistance of the building columns to horizontal movement. This resistance constitutes a relative stretching of the slabs which appears to be great enough to cause tensile failure of the concrete.

3. Based partly on this study and partly on experience with other longer buildings in our plants, we think that we have not reached a practical limit to the length of a continuous rigid frame building. The satisfactory record of several multistory buildings in our plants up to 340 ft long leads us to say that there should be no important difficulties with continuous frames up to 400 ft or even longer.

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From a symposium sponsored by ACI Committee 209

Title No. 53-44

Plastic Shrinkage*

By WILLIAM LERCH†

SYNOPSIS

Plastic shrinkage and plastic shrinkage cracking sometimes occur in the exposed surface of freshly-placed concrete. This shrinkage and cracking is caused by a rapid evaporation of water from the surface of the concrete. Conditions that determine the rate of evaporation are described. Construction procedures and practices that can minimize the causes of this type of shrinkage and cracking are recommended. Specific cases cited show how application of these procedures has solved the problem. It is believed that the recommended corrective measures will solve the problem of plastic shrinkage and plastic shrinkage cracking on construction projects.

INTRODUCTION

Plastic shrinkage is the shrinkage that occurs in the surface of fresh concrete within the first few hours after it has been placed, that is, while the concrete is still plastic and before it has attained any significant strength. This type of shrinkage is not objectionable in itself but it is sometimes accompanied by development of plastic shrinkage cracks which are unsightly and objectionable. The plastic shrinkage and plastic shrinkage cracks occur mostly on horizontal surfaces and can be practically eliminated if appropriate measures are taken during construction to minimize the causes.

IDENTIFICATION OF PLASTIC SHRINKAGE CRACKS

Plastic shrinkage cracks usually have considerable depth. On suspended floor slabs they sometimes extend through the floor. Cores taken from one pavement project have shown that the cracks extended to a depth of 4 in. They are usually almost straight line cracks without any definite symmetry or pattern, although occasionally they have been observed in a crow-foot pattern. The depth and general lack of pattern of the plastic shrinkage cracks differentiate them from craze cracks which are very shallow and form a hexagonal pattern. The plastic shrinkage cracks develop shortly after the sheen disappears from the surface of the concrete and when this type of cracking starts it can be observed to proceed very rapidly. Careful observation of the development of cracks on a construction project provides

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†Member American Concrete Institute, Head, Performance Tests Group, Research and Development Laboratories, Portland Cement Association, Chicago, Ill.

a means of differentiating plastic shrinkage cracks from the cracks that sometimes develop from other causes such as swelling of the subgrade, settlement of the formwork or contraction caused by a rapid drop in temperature.

Plastic shrinkage cracks are not usually progressive. They develop before the concrete hardens and they retain their original shape. Field surveys have shown that they do not necessarily impair the performance of pavements or structures. However, the cracks are unsightly and they should be eliminated wherever possible.

CAUSES OF PLASTIC SHRINKAGE AND CRACKING

Field investigations indicate that the principal cause of plastic shrinkage and plastic shrinkage cracking is an excessively rapid evaporation of water from the concrete surface. Even when the same materials, proportions and methods of mixing, handling, finishing, and curing are used, cracks may develop one day but not the next. This is usually due to a change in weather conditions that increases the rate of evaporation. If the rate of evaporation exceeds the rate at which bleeding water rises to the surface, then plastic shrinkage and cracking are likely to occur.

The disappearance of the sheen from the surface of the concrete indicates the time when the rate of evaporation exceeds the rate at which bleeding water rises to the surface. The time required to attain this condition will be influenced by the temperature, relative humidity, and wind velocity of the air and the temperature and bleeding characteristics of the concrete. At this stage the surface of the concrete has attained some initial rigidity; it cannot accommodate the rapid volume change of plastic shrinkage by plastic flow, and it has not developed sufficient strength to withstand tensile stress. Thus plastic shrinkage cracks may develop.

After concrete is placed, the aggregate and cement start to settle and water rises or bleeds to the surface. The rate at which this water reaches the surface and the total quantity that accumulates depend on the depth of concrete, on materials used, mix proportions, and the temperature of the concrete. The rate and duration of bleeding influence the time at which the rate of evaporation of water from the surface exceeds the rate at which bleeding water rises to the surface. However, field investigations have shown that the bleeding characteristics of the concrete do not have a major influence on plastic shrinkage and plastic shrinkage cracking. This type of cracking is not new. It has been with us for many years. Plastic shrinkage and plastic shrinkage cracking occurred many years ago when coarse ground, high bleeding cements were used. They occur with modern fine ground cements and with air-entrained concrete, both of which reduce the bleeding capacity. Furthermore, on some projects an attempt has been made to eliminate plastic shrinkage cracking by increasing the bleeding characteristics of the concrete, either by increasing the water-cement ratio (slump) or by using different cements or aggregates. These changes in the characteristics of the concrete did not have any major influence on the plastic shrinkage

or the plastic shrinkage cracking. The very high rate of evaporation (as much as 0.74 lb per sq ft per hr) that sometimes occurs on construction projects, and that greatly exceeds the bleeding rate, provides further evidence that the bleeding characteristics of the concrete cannot have a significant effect on plastic shrinkage and plastic shrinkage cracking.

RATE OF EVAPORATION

The rate of evaporation of water from the surface of the fresh concrete is the most important factor influencing plastic shrinkage and plastic shrinkage cracking. A high rate of evaporation is conducive to plastic shrinkage and plastic shrinkage cracking.

The rate of evaporation is influenced by the temperature, relative humidity, and wind velocity of the air and the temperature of the concrete. Even relatively small changes in these temperature and atmospheric conditions may have a pronounced effect on the rate of evaporation, especially if they occur simultaneously and supplement each other.

The effect of variations in concrete and air temperatures, relative humidity, and wind velocity on the drying tendency at the job site is shown in Table 1 prepared by Carl A. Menzel. These data indicate that the rate of evaporation from the surface of the fresh concrete can range from zero to as much as 0.74 lb per sq ft per hr under conditions that may be encountered on construction projects. For example, the data for Group 1, Table 1, show that as the wind velocity increases from 0 to 25 mph, the rate of evaporation increases from 0.015 to 0.135 lb per sq ft per hr, or about nine times. The data for Group 2 show that as the relative humidity is decreased from 90 percent to 10 percent, the rate of evaporation is increased from 0.020 to 0.175 lb, or over eight times. The data in Group 3 show that an increase in the concrete and air temperature from 50 to 100 F increases the rate of evaporation from 0.026 to 0.180 lb per sq ft per hr, or about seven times. On the other hand, the data in Group 4, Case No. 18, show that evaporation rate is negligible when the temperature of the concrete and the dew point of the air are equal.

Even though the air may be cool and in a saturated condition at 100 percent relative humidity, there can be a significant evaporation of water from the surface of the concrete if the temperature of the concrete is higher than the air temperature, as shown by the data in Group 5, Table 1. Thus it is possible to have plastic shrinkage in concrete and plastic shrinkage cracking in cold, damp weather. Plastic shrinkage cracking has been observed on construction projects under these conditions.

CORRECTIVE MEASURES

A few simple precautions can be taken to minimize the causes of plastic shrinkage and plastic shrinkage cracking. They should be considered in planning the construction procedures for a job or in dealing with the problem if it occurs after construction is started.

TABLE 1—EFFECT OF VARIATIONS IN CONCRETE AND AIR TEMPERATURES, RELATIVE HUMIDITY AND WIND VELOCITY ON DRYING TENDENCY OF AIR AT JOB SITE

Case No.	Concrete temperature, F	Air temperature, F	Relative humidity, percent	Dew point, F	Wind velocity, mph	Drying rate, lb per sq ft per hr
Group 1—Increase in wind velocity						
1	70	70	70	59	0	0.015
2	70	70	70	59	5	0.038
3	70	70	70	59	10	0.062
4	70	70	70	59	15	0.085
5	70	70	70	59	20	0.110
6	70	70	70	59	25	0.135
Group 2—Decrease in relative humidity						
7	70	70	90	67	10	0.020
8	70	70	70	59	10	0.062
9	70	70	50	50	10	0.100
10	70	70	30	37	10	0.135
11	70	70	10	13	10	0.175
Group 3—Increase in concrete and air temperatures						
12	50	50	70	41	10	0.026
13	60	60	70	50	10	0.043
14	70	70	70	59	10	0.062
15	80	80	70	70	10	0.077
16	90	90	70	79	10	0.110
17	100	100	70	88	10	0.180
Group 4—Concrete at 70 F; decrease in air temperature						
18	70	80	70	70	10	0.000
19	70	70	70	59	10	0.062
20	70	50	70	41	10	0.125
21	70	30	70	21	10	0.165
Group 5—Concrete at high temperature, air at 40 F, 100 percent relative humidity						
22	80	40	100	40	10	0.205
23	70	40	100	40	10	0.130
24	60	40	100	40	10	0.075
Group 6—Concrete at high temperature, air at 40 F, variable wind						
25	70	40	50	23	0	0.035
26	70	40	50	23	10	0.162
27	70	40	50	23	25	0.357
Group 7—Decrease in concrete temperature; air at 70 F						
28	80	70	50	50	10	0.175
29	70	70	50	50	10	0.100
30	60	70	50	50	10	0.045
Group 8—Concrete and air at high temperature, 10 percent relative humidity, variable wind						
31	90	90	10	26	0	0.070
32	90	90	10	26	10	0.336
33	90	90	10	26	25	0.740

Some of the corrective measures involve good concreting practices that should be applied on all construction projects. These are:

1. Dampen the subgrade and forms.
2. Dampen the aggregates if they are dry and absorptive.
3. Lower the concrete temperature in hot weather.
4. Avoid overheating the concrete in cold weather.

Further corrective measures would be required when weather conditions are such that a high rate of evaporation from the surface of the concrete may be anticipated. These corrective measures should be directed toward decreasing the time during which evaporation can occur or reducing the rate of evaporation. The following procedures have been found to be effective in field experience:

1. Reduce time between placing and the start of curing by improved construction procedures.
2. Start curing the concrete as soon as possible after placing and finishing. Apply a membrane curing compound, wet burlap or sand, light colored paper, or other curing procedures as early as possible.
3. Protect the concrete with temporary coverings or apply a fog spray during any appreciable delay between placing and finishing.
4. Erect windbreaks to reduce the wind velocity over the surface of the concrete, and
5. Provide sunshades to control the temperature at the surface of the concrete.

The corrective measures to be applied will vary on different projects. They should be selected as those most readily applicable to a particular project. A few illustrations are cited.

Plastic shrinkage cracking was observed intermittently during the first 6 weeks of construction of runway at a municipal airport. Observations on this project clearly demonstrated the relationship between weather conditions and the occurrence of plastic shrinkage cracks. On one day the temperature was 93 to 96 F, the relative humidity ranged from 30 to 45 percent and the wind velocity was 5 to 8 mph. No cracking occurred on that day. On the following day the temperature remained the same, the relative humidity was lower, 15 to 25 percent, and the wind velocity was higher, 10 to 15 mph. Plastic shrinkage cracking occurred over the entire construction of that day. A liquid membrane curing compound was being applied about 1½ to 2 hr after the final belting. Careful observation disclosed that the plastic shrinkage cracking occurred before the application of the curing compound. Some slight modifications in construction prodecures were recommended to permit an earlier application of the curing compound, and the compound was then applied immediately after the final belting. Construction on this project continued for several months to complete the job, with only an occasional plastic shrinkage crack. Similar early application of curing procedures has solved the plastic shrinkage cracking on other projects.

Somewhat different corrective measures are required on the construction of suspended floor slabs where a hard steel-trowel finish is required. Plastic shrinkage cracking has been observed on such projects before the completion of the final troweling. Obviously the early application of curing would not

correct this condition. Field investigations have shown that the application of a fog spray or a cover of wet burlap during the initial hardening prior to troweling, or between successive passes of the trowel, prevented the cracking. The use of sunshades or windbreaks should be equally suitable for this purpose.

It is believed that corrective measures cited in this paper could be applied to solve the problem of plastic shrinkage and plastic shrinkage cracking on construction projects.

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Title No. 53-45

Determination of Creep Strain of Concrete Under Sustained Compressive Stress*

By F. EUGENE SEAMAN†

SYNOPSIS

Presents the many problems involved in the measurement of creep strain in concrete as distinguished from elastic strain and other volume change factors. Describes equipment and procedures developed for the mass accumulation of reliable data necessary to evaluate the variables in concrete. Limited data indicate that curing method may greatly influence the creep strain property of concrete. Creep strain ratio values, for various concrete stresses, are plotted against time for maximum creep-time curve.

INTRODUCTION

Concrete, composed of compacted sand grains and stone particles and held together by cementing paste, is not a rigid elastic solid. The hydrated cement is described by many authorities to be in the nature of a viscous liquid or gel.[1] From this it may be inferred that extremely low stresses will in time result in inelastic strain, due to the flow of the viscous liquid, in addition to the instantaneous or elastic strain that occurs as the stress is applied. This condition of inelastic deformation, variously called plastic flow or creep strain, resulting from time-stress effect is considered to approach an asymptotic maximum value as the load is transferred from the cement paste to the aggregate particles.

On short time or relatively instantaneous stress applications, concrete of good quality exhibits a linear stress-strain relationship to a comparatively high percentage of its ultimate compressive stress.[2] Many authorities have stated that cement-rich concretes have high creep strains. Lean concrete, on the other hand, has a curved stress-strain relationship indicating strains greater than predicted from elastic consideration of the applied load. The distinction between elastic and creep strain based on whether the strain is recoverable or not, must be carefully applied since part of the strain which appears to be inelastic does recover in time. The behavior of concrete with a minimum of cement between aggregate particles does not account for the cement gel displacement theory unless it can, in part, be explained by the higher intensity of stress on the cement paste in a lean concrete.

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†Chief Research Engineer, Lock Joint Pipe Co., Research Laboratory, Wharton, N. J.

In consideration of the elastic strain and the inelastic strain it is convenient[3] to express these variables as a ratio (R = creep strain ratio).

$$R = \frac{\text{inelastic deformation}}{\text{elastic deformation}}$$

Depending upon the quality of concrete under observation, tests have shown that the creep strain can be less than the elastic strain and may be more than five times the elastic strain. The ratio R then allows an expression of creep strain in terms of the instantaneous deformation or elastic strain. Elastic strain is directly proprotional to the applied stress (Hooke's law) and with sufficient data it may be possible to show what mathematical relationship exists between inelastic creep and applied stress and other variables..

TEST EQUIPMENT

In the study of the elastic and creep strain and shrinkage deformation characteristics of concrete, it is essential to define and control the conditions under which observations will be made. The effect of variations in temperature and humidity on the dimensions of the specimens must be reduced to a minimum in order that the elastic and creep strains can be measured accurately. The stress applied to the specimen should be maintained constant throughout the observation period. A dead-weight tester in this case is ideal but the physical size of such a device places a practical limit on the number of test specimens that may be studied. A more practical method for applying a sustained load is the use of elastic spring devices in which the total strain is large compared to the creep strain of the specimen. The stored load strain can be either tensile or compressive strains in the loading device. This large difference in the respective strain magnitudes of the steel load spring strain and the creep strain of the concrete, results in a relatively small decrease in the sustained load or applied stress.

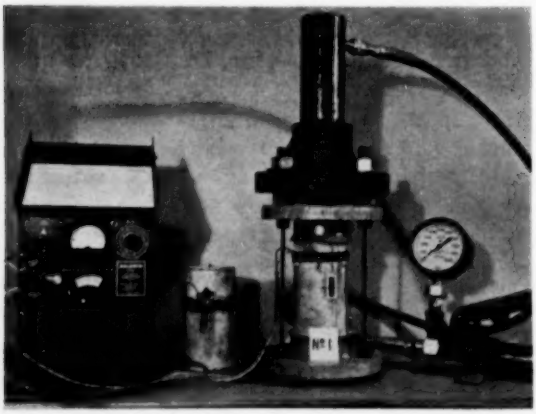

Fig. 1—Original tester

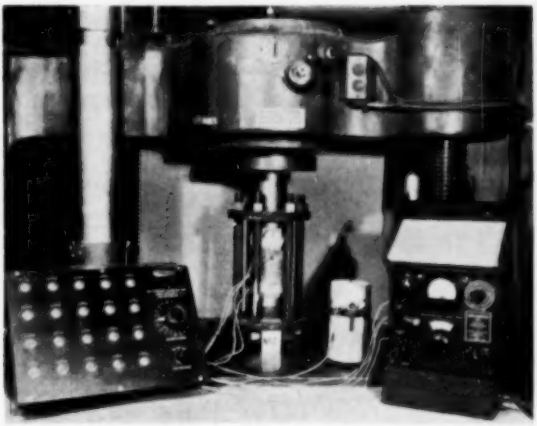

Fig. 2 — Test arrangement No. 2

The original tester and loading method and SR-4 strain measuring equipment are shown in Fig. 1. Load strain is produced by the hydraulic jack and stored in the highly stressed connecting wires. This design was the first of a series developed in a study of load-sustaining equipment. The method necessitated preloading the cylinder in a testing machine to determine the strain at test load. The hydraulic jack was then used to apply load until the same strain was produced in the cylinder. Bearing surface variations and inaccuracies in replacement resulted in many inconsistencies in loading and measuring creep strains.

A significant improvement in tester design and loading method is shown in Fig. 2. This unit is equipped with dished type compression spring washers used in pairs underneath the bottom bearing block. The washers are aligned by a pin embedded in the base plate and projecting into a clearance hole in the bearing block. The top bearing block is loaded through a 1-in. steel ball to produce true axial loading on the cylinder. Load is applied by the testing machine to the tester through a cylindrical block bearing on the top end plate. At test load the bolts are tightened and the stress is gradually transferred from the machine to the bolts. This method has proven satisfactory except for the difficulty in getting the bars evenly stressed.

Additional refinements made to the tester and loading method are illustrated in Fig. 3. The number of connecting rods has been reduced by using higher strength steel. The bottom bearing block is fixed to the base plate and the top bearing block is loaded axially through both the compression spring washers and 1-in. steel ball. The loading method utilizes a special frame to stress the cylinder and connecting rods simultaneously. The tester is hung in the frame by the top ends of the connecting rods. The machine load is applied to the top end plate of the tester by a cylindrical bearing block which fits through a hole in the center of the framehead. When the test load is reached, the connecting rod nuts are turned down against the upper end plate and the machine load removed. Strain measurements are

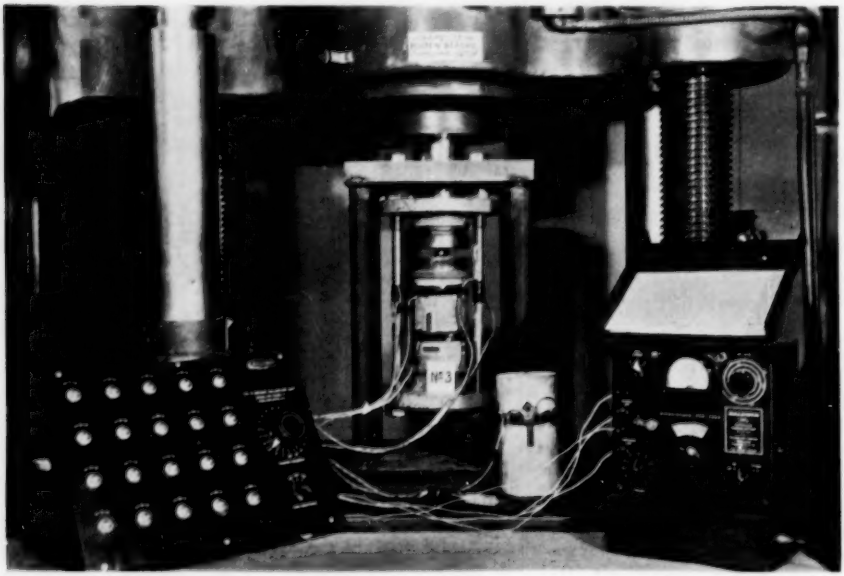

Fig. 3—Test arrangement No. 3

made before and after the nuts are turned down. The tester is removed from the frame simply by lifting out the cylindrical bearing block and removing the top nuts.

A detailed view of the latest tester is shown in Fig. 4. It is fundamentally the same as the tester in Fig. 3, except the compression spring washers are on the bottom. This was done to produce pure axial compression in the cylinder and to overcome the effect of uneven stresses in the connecting rods.

STRAIN MEASURING PROCEDURES

Strain can be measured accurately by sensitive mechanical or electrical resistance strain gages. The former are probably more dependable but the high individual cost and time consuming installation severely limits the number of tests that may be performed. The paper base electrical resistance strain gage is simple and with proper installation procedures will provide satisfactory data for observation periods of 3 to 6 months. For longer periods mechanical gages or other means have been used to avoid the bonding failures of cellulose cements. The simplicity of resistance gages permits multiple installations and is therefore more applicable to a study requiring large numbers of gages and many readings.

It can be shown that creep behavior of concrete exposed to varying humidity and temperatures, over a long time, is much the same as concrete under controlled conditions of temperature and moisture. Thus the elastic strain and inelastic or creep strain can be measured and separated—providing

dimensional changes, due to temperature and moisture variations, are known or not allowed to occur. Two methods of study that can be used to evaluate the creep of concrete are:

1. The behavior of carefully made prestressed concrete product specimens in which the theoretical (assuming elastic materials) behavior is compared with the actual performance; and with corrections for temperature and moisture, the inelastic strains may be evaluated.

2. Laboratory specimens in which the humidity, temperature, and other variables affecting the dimensional stability of the concrete can be accurately controlled, thus permitting direct measurement of the creep strain.

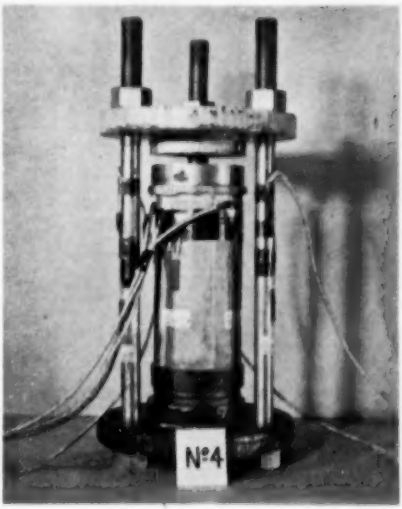

Fig. 4—Latest tester

In the design of prestressed concrete products it is necessary that the creep strain properties of concrete be known for stress loadings at a comparatively early age. The effect of accelerated curing of prestressed concrete with respect to the creep strain magnitude must also be studied as it relates to the design of the product.

Prestressed concrete products are usually fully loaded or stressed at an age of 4 days to 2 weeks so that manufacturing and handling economies may be accomplished. The creep strain behavior of concrete must, therefore, be carefully studied at this age and its effect on the performance of the product evaluated. The effect of moisture shrinkage on the early age concrete specimen thus becomes difficult to separate from the creep strain due to sustained stresses.

DISCUSSION

In the determination of creep strain in concrete, the different creep rates of the cement-rich areas and aggregates must be considered. It has been shown that a cement-rich area in concrete has a different strain rate than an area containing large aggregate particles. For example, the surface of a concrete specimen made with a crushed granite stone and manufactured sand was cut off with a masonry saw and the newly exposed surface tested for moduli of elasticity. Polished coarse aggregate surfaces had a modulus of elasticity of 7.8×10^6 psi and the cement-rich area, 2.8×10^6 psi. The over-all modulus of elasticity for the specimen was found to be 3.6×10^6 psi. For these tests no measurable creep strain was noted in the stone particle, while the cement-rich area exhibited a creep ratio of 0.2 of the elastic strain on a short test of 9 hr. This test illustrates that the modulus of elasticity of concrete is dependent on the moduli of the cement and the

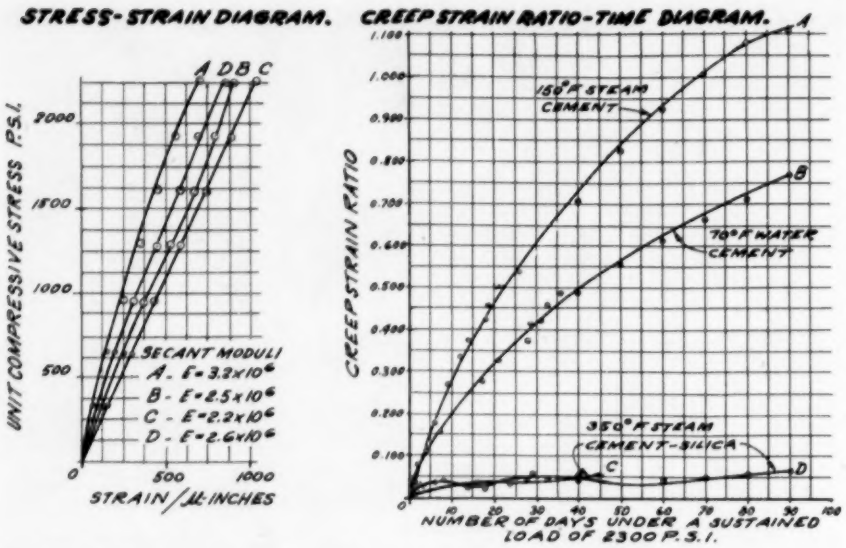
STRESS-STRAIN DIAGRAM.
CREEP STRAIN RATIO-TIME DIAGRAM.
UNIT COMPRESSIVE STRESS P.S.I.
STRAIN / IN. INCHES
A D B C
SECANT MODULI
A - E = 3.2 x 10⁶
B - E = 2.5 x 10⁶
C - E = 2.2 x 10⁶
D - E = 2.6 x 10⁶
CREEP STRAIN RATIO
150° F STEAM CURED CEMENT
70° F WATER CURED CEMENT
350° F STEAM CURED CEMENT-SILICA
NUMBER OF DAYS UNDER A SUSTAINED LOAD OF 2300 P.S.I.

Fig. 5—Comparison of elastic and creep strain properties of cement and cement-silica, 4 x 8-in. cylinders as affected by curing conditions

aggregate particles, while the creep of concrete is dependent on the creep of cement paste, assuming the aggregates are of good quality.

Special 4 x 8-in. cylinders composed of portland cement and silica flour (100 percent—No. 200 mesh sieve), with no aggregate, were subjected to autoclave curing conditions. They were found to have comparatively low moduli of elasticity and almost no creep strain under 2300 psi sustained loading for periods up to 90 days. Atmospheric pressure steam-cured and water-cured cylinders of pure portland cement had comparable moduli of elasticity but the creep strain ratios were 18 and 12 times greater, respectively, at the same age. It should be emphasized that the low creep strain obtained by autoclave curing was confirmed by a second cylinder for a load period of 45 days. See Fig. 5 and Table 1 for curves and compilation of data. A

TABLE 1—EFFECT OF CURING CONDITIONS ON STRENGTH, ELASTICITY, AND CREEP CHARACTERISTICS OF 4 x 8-IN. CEMENT AND CEMENT-SILICA CYLINDERS

Code	Materials	Curing cycles	Age at loading, days	Compressive strength, psi	Secant modulus of elasticity $\times 10^6$	Time under sustained 2300-psi load, days	Creep strain ratio
A	Cement	48 hr moist 70 F 72 hr steam 150 F	9	9,350	3.2	90	1.13
B	Cement	7 days moist 70 F 14 days water 70 F	30	8,550	2.5	90	0.78
C	Cement-silica	24 hr moist 70 F 16 hr steam 350 F	70	13,650*	2.2	45	0.06
D	Cement-silica	24 hr moist 70 F 16 hr steam 350 F	30	12,850*	2.6	90	0.06

*Localized surface failures at approximately 8000 psi.

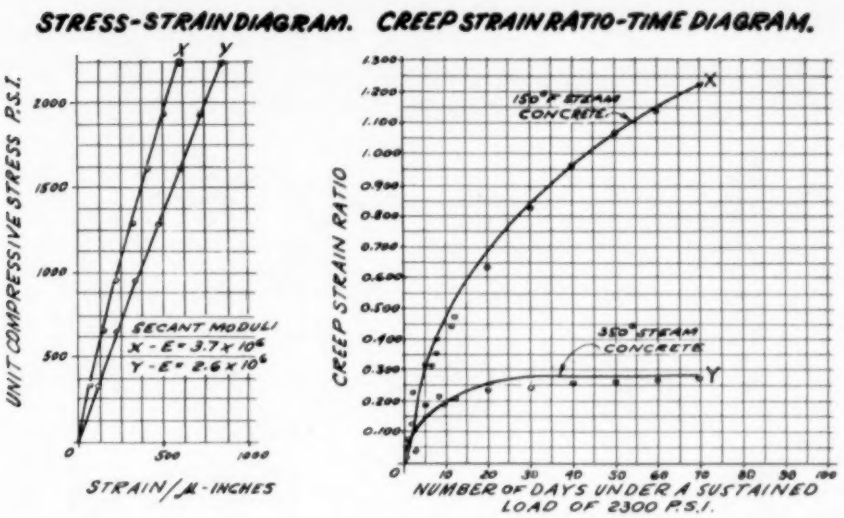
STRESS-STRAIN DIAGRAM.
CREEP STRAIN RATIO-TIME DIAGRAM.
UNIT COMPRESSIVE STRESS P.S.I.
STRAIN/μ-INCHES
SECANT MODULI
X - E = 3.7 x 10⁶
Y - E = 2.6 x 10⁶
CREEP STRAIN RATIO
150° F STEAM CONCRETE
350° STEAM CONCRETE
NUMBER OF DAYS UNDER A SUSTAINED LOAD OF 2300 P.S.I.

Fig. 6—Comparison of elastic and creep strain properties of 4 x 8-in. concrete cylinders as affected by curing conditions

comparison between autoclave and atmospheric steam-curing conditions was also made using 4 x 8-in. normal portland cement concrete cylinders. The autoclaved cylinder had a 30 percent lower modulus of elasticity and a creep strain ratio of one one-fifth the atmospheric steam-cured cylinder (data in Fig. 6 and Table 2).

These tests indicate that creep strain in concrete under sustained stress is fundamentally related to the state of cure of the cement gel formation. In other words, if the cement hydration can become sufficiently complete and stable before the concrete is subjected to stress, the creep strain behavior will approach that found in durable hard aggregate particles.

Creep strain ratios may be conveniently obtained by hydrostatically testing prestressed concrete pipe and by comparing the theoretical (assuming elastic materials and constant moisture conditions) wall strain behavior with the observed wall strain. The difference, other dimensional variations being corrected or eliminated, is attributed to the creep strain that occurred in the pipe wall due to the prestressing force. Fig. 7 is a graph showing the results of a large number of experimental pipe subjected to these hydrostatic tests.

TABLE 2—EFFECT OF CURING CONDITIONS ON STRENGTH, ELASTICITY, AND CREEP CHARACTERISTICS OF 4 x 8-IN. CONCRETE CYLINDERS

Code	Mix proportions by weight	Curing cycles	Age at loading, days	Compressive strength, psi	Secant modulus of elasticity $\times 10^6$	Time under sustained 2300-psi load, days	Creep strain ratio
X	1:1.9:2.9	16 hr steam 100 F 72 hr steam 150 F	10	5670	3.7	70	1.22
Y	1:1.9:2.9	24 hr moist 70 F 8 hr steam 350 F	10	5630	2.6	70	0.27

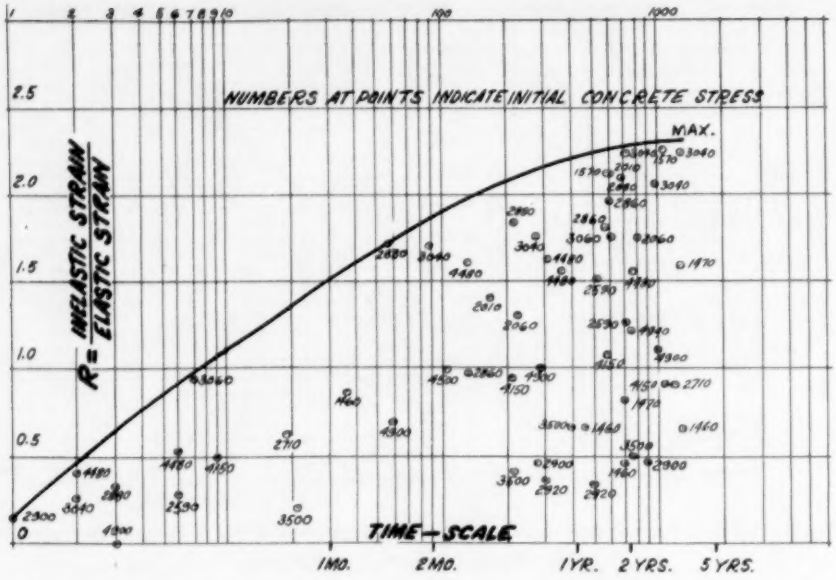

Fig. 7—Concrete creep versus time—experimental prestressed concrete structures

SUMMATION

A review of technical literature on concrete shows a real need for reliable information on the creep strain behavior of concrete. To obtain this data, a satisfactory and economical method must be developed for mass accumulation of test data to evaluate the many variables that enter into making of concrete. Creep strain deformation in concrete as differentiated from recoverable drying shrinkage is important in prestressed concrete design. A full understanding of its magnitude for conditions of exposure under which the prestressed product is used would greatly simplify the design of such products.

Improved curing methods are important in reducing creep strain characteristics of concrete. A satisfactory method of determining full potential creep strain in a short time, say 28 days, would eliminate empirical loss factors in concrete products design.

REFERENCES

1. Thomas, F. G., "Creep of Concrete Under Load," International Association of Testing Materials, 1937.

2. Seaman, F. E., "Tests of Prestressed Concrete Cylinder Pipe," *Journal*, American Water Works Association, V. 42, No. 11, Nov. 1950, pp. 1065-1082.

3. Komendant, August E., *Prestressed Concrete Structures*, McGraw-Hill Book Co., Inc., New York, 1952, p. 45.

Discussion of this paper should reach ACI headquarters in triplicate by June 1, 1957, for publication in the Part 2, December 1957 JOURNAL.

CURRENT REVIEWS

of Significant Contributions in Foreign and Domestic Publications

Bridges

Concrete bridge decks directly exposed to traffic (Unmittelbar befahrene Brueckenfahrbahnplatten aus Beton)

FRIEDRICH PFAUE, *Der Bauingenieur* (Berlin), V. 31, No. 1, Jan. 1956, pp. 1-16

Reviewed by ARON L. MIRSKY

Prestressing and composite construction having made possible jointless bridge slabs, these slabs may now be left exposed, their top surfaces acting as the wearing surface, rather than used as the base for asphalt, as had been the European practice. Author examines production of concrete deck slabs which will have a good finish and be dense and strong. Results of two test slabs are given to show it can be done, using vibrating-screed paving machines wide enough to pave the entire deck width in one pass.

Bridge construction (Brueckenbau)

FRITZ LEONHARDT, *VDI Zeitschrift* (Duesseldorf) V. 98, No. 17, June 11, 1956, pp. 944-952

Reviewed by ARON L. MIRSKY

This lengthy survey article, one of a series of annual reviews covering entire field of engineering, covers both steel (nominally pp. 944-949) and concrete (pp. 949-952) structures. But the steel bridges are in many cases distinguished by the composite concrete decks, which in turn are distinguished by prestressing of one type or another, so that concrete plays a prominent role indeed.

Prestressing restores weakened truss bridge in Birmingham, England

P. S. A. BERRIDGE and D. H. LEE, *Civil Engineering*, V. 26, No. 9, Sept. 1956, pp. 48-49

Describes the restoration of a structural steel truss girder bridge by converting it into a prestressed composite structural steel and concrete structure. Post-tensioned Stressteel bars were used to apply compression in the bottom chord. After prestressing, the rods and the bottom chord were encased in concrete.

Road bridges in Norway

ARNULF ARILD, *The Structural Engineer* (London), V. 34, No. 4, Apr. 1956, pp. 115-135

Reviewed by C. P. SEISS

Reviews development of highway bridges in Norway during the last 150 years. Describes recent bridges, including a composite welded plate girder with temporary and pemanent prestressing cables, a 603-ft span steel arch, and a two-span suspension bridge. Also describes Norwegian design loadings.

Precast concrete bridge decks

Constructional Review (Sydney), V. 29, No. 8, Aug. 1956, pp. 30-36

Describes many of the systems used for precast concrete bridge deck units by the various highway departments in the United States and also some English developments. A good general survey.

Construction

Port development works at Colombo, Ceylon

The Engineer (London), V. 201, No. 5238, June 15, 1956, pp. 678-681

Reviewed by ARON L. MIRSKY

General description of new additions to harbor facilities, comprising over 10,000 ft of deep-water quays and other structures. Of especial interest are the quay walls built of sloping precast concrete blocks.

A part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 8, Feb. 1957, *Proceedings* V. 53. Address P. O. Box 4754, Redford Station, Detroit 19, Mich. Where the English title only is given in a review, the book or article reviewed is in English. If it is followed by a foreign title the work reviewed is in that language. In those cases where the foreign title cannot conveniently be set in type or is not available, the language of the original article is indicated in parentheses following the English title. Copies of articles or books are **not** available through ACI. Available addresses of publishers are listed in the June "Current Reviews" each year. In most cases ACI can furnish addresses of publications added later.

For those members that cut apart this section for pasting on cards for card indexes, a limited number of complimentary reprints of the "Current Reviews" section are available from ACI headquarters on request.

Consideration of the construction temperature and the extreme temperature limits in the construction of expansion joints (Berucksichtigung der Entstehungstemperatur und der aeussersten Temperaturgrenzen bei der Konstruktion von Bewegungsfugen)

WOLFGANG STIEDA, *Der Bauingenieur* (Berlin), V. 30, No. 8, Aug. 1955, pp. 285-286
Reviewed by ARON L. MIRSKY

Considers several cases of damage to structures caused by lack of consideration of temperature during construction in relation to maximum (summer) and minimum (winter) temperatures.

Big batch trucks set pace for concrete paving crews

R. MONSON, *Contractors and Engineers*, V. 53, No. 9, Sept. 1956, pp. 6-13

Describes construction methods used on the 55-mile Kansas Turnpike, a divided superhighway completed in one construction season. Covers all operations from grading through final curing and sawing joints in the concrete.

Design and construction of the printing works at Debden

HOWARD ROBERTSON, OVE ARUP, R. S. JENKINS, and H. F. ROSEVEAR, *The Structural Engineer* (London), V. 34, No. 4, Apr. 1956, pp. 137-151
Reviewed by C. P. SIESS

A complete structure involving many interesting design and construction features, including: a precast retaining wall; precast multistory construction; flat slab and flat plate floors, the latter constructed with traveling forms; prestressed arches, partly precast, supporting a shell roof; prestressed double-cantilever girders supporting a shell roof; precast wall sections; and combination of direct and inverted shells.

Dome is built on ground, then earth is excavated for municipal auditorium

R. DAY, *Contractors and Engineers*, V. 53, No. 9, Sept. 1956, pp. 88-91

The 217-ft diameter reinforced concrete dome at Albuquerque, N. M., for its new civic auditorium, used an unusual order of construction. It consisted of first building 22 individual separate footings and pilasters to support the dome; backfilling; shaping all excavated fill plus additional fill into the earth form for the concrete dome. After placing, the perimeter beam was prestressed and the interior excavated. The general contractor reports a saving of approximately $50,000 total on a $1,100,000 structure due to the unusual procedure.

Reconstruction of the Gallions Lower Entrance Lock at the Royal Docks of the Port of London Authority

JOHN A. FISHER, *Proceedings*, Institution of Civil Engineers (London), Part II, V. 5, No. 2, June 1956, pp. 136-169 (including discussion)
Reviewed by ARON L. MIRSKY

Description of remodeling of lock, opened in 1886, to accommodate modern shipping. Some interesting demolition and reconstruction work was involved.

Construction of the Koblenz-Oberwerth outdoor swimming pool (Der Bau des Freischwimmbades Koblenz-Oberwerth)

HANS HELM, *Der Bauingenieur* (Berlin), V. 30, No. 8, Aug. 1955, pp. 286-290
Reviewed by ARON L. MIRSKY

Comments on design and construction of the basin, diving tower and platform, water-purification facility, and spectator grandstand.

Long-span hall (Eine weitgespannte Shedhalle)

GEORG KIRCHNER, *Der Bauingenieur* (Berlin), V. 30, No. 11, Nov. 1955, pp. 405-407
Reviewed by ARON L. MIRSKY

Design and construction of an industrial structure part of whose roof, with a clear span of approximately 52 ft, is composed of two shell roof segments, with "north light" glazing, in series.

New type of floor slab construction in steel frame construction (Neuartige Deckenkonstruktion im Stahlskelettbau)

F. HERKE, *Der Bauingenieur* (Berlin), V. 30, No. 8, Aug. 1955, pp. 299-301
Reviewed by ARON L. MIRSKY

Description of a lightweight floor-suspended ceiling system applicable to multistoried steel frame construction. Small square precast concrete slabs are laid on light-gage steel members and protruding bar ends welded together and to the steel members. An insulating layer, finishing course, and wear-

ing surface are added later. The ceiling members are suspended from hangers attached to the steel members, so that sound transmission is reduced.

Dams

Developments in Swiss dam construction (Entwicklungen im schweizerischen Talsperrenbau)

GEROLD SCHNITTER, *Der Bauingenieur* (Berlin), V. 30, No. 10, Oct. 1955, pp. 363-367
Reviewed by ARON L. MIRSKY

Swiss water power facilities have undergone great expansion since the World War II, and this has, of course, led to improvements in design and construction. Besides describing and comparing various dams, article devotes considerable attention to progress in cooling of mass concrete.

Construction of the Jablanica reservoir, the first water power facility in the Neretva river system in Yugoslavia (in German)

DRAGAN CARIC, *Der Bauingenieur* (Berlin), V. 30, No. 6, June 1955, pp. 201-207
Reviewed by ARON L. MIRSKY

Describes dam, which is a gravity arch type with a ski-jump overflow and was designed by the trial-load method, the underground power station, and the appurtenances.

Design

Exact analysis of skewed grid (girder-and-floor beam) bridges with the aid of influence lines (Genaue Berechnung schiefer Traegerrostbruecken mit Einflusslinien)

H. BEER and F. REISINGER, *Der Bauingenieur* (Berlin), V. 30, No. 12, Dec. 1955, pp. 425-431
Reviewed by ARON L. MIRSKY

Method of deformations is used to obtain exact solution of stringer-and-diaphragm or girder-and-floor beam skew bridges, including effect of deck stiffness.

Homberg (*Kreuzwerke*, Berlin, 1951) had suggested transforming skew bridge into equivalent square bridge for simpler calculations, bridges to have equal spans and girder spacings but with floorbeam stiffnesses changed correspondingly. Analysis of a four-stringer, three-diaphragm bridge for 30-, 45-, 60-deg skews shows that use of equivalent square bridge gives moments respectively 3, 10, and 23 percent larger than are found by using the true skew bridge.

Computations of bending, torsion, and shear are given for the structure described above.

Present situation on retaining wall design

P. W. ROWE, *The Structural Engineer* (London), V. 34, No. 6, June 1956, pp. 204-217
Reviewed by C. P. SIESS

Discusses design of various types of earth-retaining structures in terms of code requirements, modern earth-pressure theories, and field conditions. Considers various design criteria relating to working stresses and limit design in various combinations with loading conditions. Recommends factors of safety. Emphasis is on design of flexible walls (sheet-pile bulkheads).

Critical loads of tall building frames (Part III)

R. E. BOWLES and W. MERCHANT, *The Structural Engineer* (London), V. 34, No. 9, Sept. 1956, pp. 324-329
Reviewed by C. P. SIESS

Deals with lateral stability of a two-bay, five-story frame. Also considers stability about longitudinal axis when sway is prevented. Results compared with tests on an elastic-bar model.

Concrete and structural form

PIER LUIGI NERVI, *The Structural Engineer* (London), V. 34, No. 5, May 1956, pp. 155-172
Reviewed by C. P. SIESS

An illustrated discussion on design and construction of Nervi's unique works.

Membrane roofs for low cost housing

S. SIVASWAMI, *The Indian Concrete Journal* (Bombay), V. 30, No. 1, Jan. 1956, pp. 27-33

A comparative study of membranes—catenary, circular, and elliptical—particularly for roof construction for low-cost housing. Some examples of floor construction are also included. The article summarizes the membrane theory, including analysis for dead load, snow load, wind load, spandrel filling load (for floors), and edge beams. Examples are given of standard design of low-cost, one-

story houses with inverted catenary-shaped curves, and conoids for either cast-in-place, shotcrete, or precast construction. An example is also included of a two-story house with membrane roof and floor for cast-in-place or precast construction.

Contribution to the analysis of isotropic, circular-cylindrical shell roofs (Ein Beitrag zur Berechnung isotroper, kreiszylindrischer Schalendaecher)

Rudolf Aschenbrenner and Hans A. Lange, *Der Bauingenieur* (Berlin), V. 30, No. 5, May 1955, pp. 171-174

Reviewed by Aron L. Mirsky

Presents tables of integration constants for shells of two types intermediate between those studied by Aas-Jakobsen (*Bauingenieur*, V. 20, 1939, pp. 394 *ff*). Six such structures constructed in Argentina are listed. Integration constants involved arise in solution of boundary problem.

Problem of the edge disturbances in circular-cylindrical shells of intermediate type (El problema de las perturbaciones de borde en cascaras cilindricas circulares de tipo intermedio)

Rodolfo Aschenbrenner and Juan A. Lange, *Tecnica* (San Miguel de Tucuman, Argentina), V. 2, No. 1, Oct. 1954, pp. 3-10

Contains much the same material as paper by same authors reviewed above, but presentation is somewhat different.

On the edge condition in solutions of plates by difference methods (Zur Randbedingung bei Scheibenloesungen mit Differenzen)

Walter Mudrak, *Der Bauingenieur* (Berlin), V. 31, No. 1, Jan. 1956, pp. 20-21

Reviewed by Aron L. Mirsky

Analytical solution using a rectangular net.

Silos (Silos—Traité Théoretique et Pratique)

Marcel Reimbert and Andre Reimbert, Éditions Eyrolles, Paris, 1956, 280 pp., 3050 F.

Presents theoretical and experimental data for the design of silos used to store grain and other materials. Consideration is given to methods of operation and the effect of filling and emptying. After discussion of the numerous tests reported, design methods are developed for the walls of silos of various shapes and for other parts of the structure.

Profuse illustrations show construction of silos of various materials with particular emphasis on reinforced concrete. Precast, prestressed, and sliding-form construction are included, also some traditional form systems.

Analysis of interconnected bridge girders by the distribution of harmonics

Leslie G. Jaeger and Arnold W. Hendry, *The Structural Engineer* (London), V. 34, No. 7, July 1956, pp. 241-266

Authors' Summary

Describes a new method for the analysis of simply supported interconnected bridge girders. The transverse system is considered to be a uniform medium of total moment of inertia equal to that of the actual transverse system and a differential equation including terms due to rotation and twist is written down for each longitudinal. These equations are solved by harmonic analysis and distribution coefficients are derived giving the amplitudes of the harmonics of the bending moment or deflection curve for each girder. The theory presented allows for any degree of torsional stiffness of the longitudinals and in the most general case the distribution coefficients are functions of three dimensionless parameters; the first is a measure of the transverse stiffness of the bridge, the second of the torsional stiffness, and the third expresses the ratio of the inertias of the outer to the inner longitudinals. For design purposes the cases of zero and infinite torsional stiffness are of greatest interest, but intermediate cases can be obtained by use of an interpolation function. An alternative and simple method is developed for zero torsional stiffness and the calculation of transverse moments is also dealt with. A number of comparisons with experimental results are quoted and the application to the design of steel and concrete bridges is demonstrated.

Calculation of the earth pressure on a vertical wall (Eine Berechnung des Erddrucks gegen eine lotrechte Wand)

Yoshichika Nishida, *Der Bauingenieur* (Berlin), V. 30, No. 9, Sept. 1955, pp. 320-322

Reviewed by Aron L. Mirsky

Theoretical solution of the problem of earth pressure and its distribution on a vertical wall with level, cohesionless backfill. Solution, based on theory of elasticity, involves Bessel functions.

Prestressed continuous beams and frames

P. B. Morice and H. E. Lewis, *Proceedings*, ASCE, V. 82, ST5, Sept. 1956, 1055-1 to 1055-29

Describes design methods for continuous prestressed beams and frames. Location of cable profiles is discussed. Concludes that the comparison between simply supported and continuous construction shows that continuity is advantageous only when large dead loads occur; also that the adoption of alternating long and short spans does not lead to economy.

Hyperbolic paraboloids and other shells of double curvature

A. L. Parme, *Proceedings*, ASCE, V. 82, ST5, Sept. 1956, pp. 1057-1 to 1057-32

Derives and illustrates the use of design formulas for doubly curved shells, including the problem of edge beams and other supports. Includes tables and curves facilitating the solution of formulas presented. Numerical examples of the solutions of two types of such shells are included.

Pavements

The Oxton by-pass extension

Reginald A. Kidd, *Proceedings*, Institution of Civil Engineers (London), Part II, V. 5, No. 2, June 1956, pp. 200-240 (including discussion)
Reviewed by Aron L. Mirsky

A short (3/4 mile) extension of the experimental road at Oxton, Nottinghamshire (*Proceedings*, Part II, V. 4, No. 1, Feb. 1955, pp. 137-166; "Current Reviews," ACI Journal, Dec. 1955, *Proc.* V. 52, pp. 498-499) was built primarily to gain experience in mechanized construction of high-quality concrete highways, in view of the great backlog of highway construction facing Great Britain. The road was designed and constructed to check the standard of construction which could and would be achieved, by an experienced contractor, in terms of riding quality, soundness of construction, joint construction, and utilization of paving plant now occupied in airfield construction. To accomplish all this in the 3/4 mile length, economic considerations had to be sacrificed. Results, while not ideal, were generally good.

Although construction methods used are stated to be based on the best current European practice, a cursory inspection indicates a decided similarity to American practice. Can it be that Europeans and Americans are starting to think alike—in certain fields at least?

Precast Concrete

Technical, economic and legal significance of construction standards and costs (Technische, wirtschaftliche und rechtliche Bedeuting der Baunormen under der technischen Baubestimmungen)

B. Wedler, *Betonstein Zeitung* (Wiesbaden), V. 21, No. 9, Sept. 1955, pp. 409-411

Quality control of concrete blocks (Der Guetestand von Mauersteinen)

G. Gottschlich, *Betonstein Zeitung* (Wiesbaden), V. 21, No. 9, Sept. 1955, pp. 412-418
Reviewed by Werner H. Gumpertz

Extensive difficulties have been experienced with the low quality of many concrete block in Germany. These two articles discuss the latest standards and codes, as well as the testing carried out by state authorities. Such tests show frequent failure of concrete block to comply with applicable standards and codes. The present efforts of the block industry for self-regulation are slowly improving this situation.

Handling methods speed placement of hangar girders

Contractors and Engineers, V. 53, No. 9, Sept. 1956, pp. 98-100

Describes construction and erection of 136-ft span prestressed girders weighing 84 tons each, which are believed to be the longest concrete beams ever precast. The beams were precast on the side, moved horizontally on rails to the hangar position, and lifted by three cranes to their final position in the roof.

Precasting speeds erection of California marine terminal

World Construction, V. 9, No. 9, Sept. 1956, pp. 26-29

Marine terminal for Union Oil Co., Oleum, Calif., is a two-story causeway, 1500 ft long, 22 ft wide, connected to a shipping wharf 1250 x 136 ft. The structure consists essentially of precast pile trestles floored with precast slabs on cast-in-place bents. Precast

concrete piles, 18 x 18 in. and 20 x 20 in., averaging 106 ft in length, were used. The entire structure is designed to resist the impact of berthing vessels. A cast-in-place topping 6 in. deep on 6-in. precast slabs is used to create a composite deck which will act like a completely monolithic unit.

Modern curing of concrete block

M. H. Gross, *Pit and Quarry*, V. 48, No. 9, Mar. 1956, pp. 223-226, 234

Application of thermodynamic principles to secure proper conditions of moisture at elevated temperatures for accelerated curing of concrete block at atmospheric pressure.

High-production precasting puts bridge ahead of schedule

A. N. Mavroudis, *Contractors and Engineers*, V. 53, No. 9, Sept. 1956, pp. 44-48

In construction of Gandy Bridge across Tampa Bay, Fla., mass production methods were used to produce the 1644 precast prestressed girders and the 1500 composite concrete piles, as well as hundreds of small concrete diaphragms. The bridge consists of precast piles driven to a 4-ft penetration in bedrock, and precast prestressed stringers covered by a cast-in-place deck slab. The floating concrete plant and erection of precast elements are described.

General

Current trends in the specification of structural safety

Alfred G. Pugsley, *The Engineer* (London), V. 201, No. 5236, June 1, 1956, pp. 595-596
Reviewed by Aron L. Mirsky

Interesting paper delivered at recent Cement and Concrete Association symposium on the strength of concrete structures. Five trends are briefly discussed in historical order: ultimate strength; probability relations between safety and strength (external loads and internal strength); design (useful, economic) life; relation between safety and economic aspects of failure; and, finally, the relation between safety and the human-risk aspects of failure. Last item involves, as author points out, the study of "both the accident psychology of the users of structures and the moral conscientiousness of structural designers." Reviewer would add that thought of "progress" as represented by expressing "human risk in economic terms" is repugnant to most engineers.

Techniques of plant maintenance and engineering—1956

Clapp & Poliak, Inc., New York, 1956, 248 pp., $10

Reports proceedings of the technical sessions held concurrently with the Seventh National Plant Maintenance and Engineering Show. Emphasis is on the problems of planning, budgeting, and supervision of maintenance of machinery, plants, and grounds. Little technical information is presented on important uses of concrete in these fields.

Simple devices for capping compression test samples

S. K. Waldorf, *ASTM Bulletin*, No. 216, Sept. 1956, pp. 67-68

Describes the use of and pictures several devices for capping compression test specimens of concrete, compacted soil, and similar materials.

Crushing and mixing under controlled conditions

M. S. Frenkel, *Engineering* (London), V. 182, No. 4714, July 13, 1956, pp. 41-42
Reviewed by Aron L. Mirsky

Describes principle and operating features of a continuous crusher and mixer. Design produces uniform particle size and uniform mixing, reduces bearing loads, minimizes wear. Machine in principle "consists of two relatively-rotating co-axial components with an annular space between them, and having operating surfaces of a certain complementary configuration."

Influence of compression and shearing strains in soil foundations on structures under earth embankments

M. G. Spangler, *Bulletin* No. 125, Highway Research Board, Sept. 1956, pp. 170-177

Discusses the effect of settlements and outward movements of embankments upon the performance of structures, such as culverts and sluiceways through embankments. The amount of settlement and lengthening of 46 culverts and sluiceways are recorded in the paper. Conclusions useful for planning such culverts are given.